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ИЗВЕСТИЯ

НАЦИОНАЛЬНОЙ АКАДЕМИИ НАУК
РЕСПУБЛИКИ КАЗАХСТАН
Қазақстан Республикасының
Ғылым Академиясының
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Қазақстан Республикасы Ұлттық ғылым академиясы "ҚР ҰҒА Хабарлары. Физикалық-математикалық сериясы" ғылыми журналының Web of Science-тің жаңаланған нұсқасы Emerging Sources Citation Index-те индекстелуге қабылданғанын хабарлайды. Бұл индекстелу барысында Clarivate Analytics компаниясы журналды одан әрі the Science Citation Index Expanded, the Social Sciences Citation Index және the Arts & Humanities Citation Index-ке қабылдау мәселесін қарастыруда. Web of Science зерттеушілер, авторлар, баспашылар мен мекемелерге контент тереңдігі мен сапасын ұсынады. ҚР ҰҒА Хабарлары. Химия және технология сериясы Emerging Sources Citation Index-ке енуі біздің қоғамдастық үшін ең өзекті және беделді химиялық ғылымдар бойынша контентке адалдығымызды білдіреді.

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THE FRACTAL SCALE-INVARIANT STRUCTURE OF A TEMPORAL HIERARCHY IN THE RELAXATION PROCESSES

Abstract. The phenomena of elastic aftereffects during loading/unloading of viscoelastic and capillary-porous bodies, relaxation of their stresses is accompanied by the energy accumulation and dissipation to be taken into account in the theory of oscillations which also considers the behavior of materials when the force is applied to them, the elastic aftereffect and stress relaxation forms ostensibly opposite energy processes that's why the main problem to one is to understand and discovery laws for such aftereffects. The goal of the research to show that the distribution of relaxation time in viscoelastic and capillary-porous media may have a scale-invariant structure and that the indirect confirmation of the scale invariance of relaxation time hierarchy can be the principle of temperature-time superposition according to which the experimental relaxation functions obtained for different temperatures can be combined with each other using the appropriate coordinate axes stretching. We used methods of viscoelastic theory, fractal analysis and methods of mathematical physics. So, in this paper, an attempt has been made to harmonize both these theories and numerous experiments on the destruction of materials described in the academic literature. It is shown that the hierarchy of times determining shear and bulk relaxation in viscoelastic/capillary-porous medium has a fractal structure and it was observed that the presence of time fractality eases the modeling of viscoelastic/capillary-porous bodies resulting in the universal relaxation function of a rather simple kind.

Key words. Aftereffect, internal friction, viscoelasticity, mechanical models, elongation ratio.

Introduction. Hereditary properties of materials have long been studied by experts. For example, faced in 1920s with the fact of elastic aftereffects, the academician A.F. Ioffe [1] described the above phenomenon as follows: "... the result of the effect of this force on the body does not manifest itself entirely at once. For a long time, exposed to a constant force, bending, twisting, tensile it continues becoming gradually weaker. Each effect leaves a trace which can be noticed after a long period of time as the reasons of its emergence disappear. There is something similar to the body's memory experienced in the past". In test machines, the load decrease and keeping the deformation unchangeable is made automatically with the help of special electronic equipment. The water is used as load in such tests. This allows a smooth stress decrease [2].

Turning to researches for energy dissipation under load, i.e., for the theory of internal friction, it can be found that some theories are based on the dependence of oscillations friction on their velocity, other theories are based on the amplitude. Some research papers are based on the M.M. Davidenkov [3] hypothesis. According to it, the energy in the conditions of oscillations application depends on the amplitude and does not depend on the velocity [4]. E.S. Sorokin [5], author of one of these hypotheses, carries out a detailed analysis of research papers on these issues. On the basis of his table containing the comparative characteristics of various theories, he made an important remark that the theory of internal friction is poorly consistent with theories describing the hereditary properties of the materials [6-9].

Moreover, the following tendency is observed: the better a theory reflects hereditary properties, the worse this theory is adapted to describe energy losses due to oscillations.

The goal of the research to show that the distribution of relaxation time in viscoelastic and capillary-porous media may have a scale-invariant (fractal) structure and that the indirect confirmation of the scale invariance of relaxation time hierarchy can be the principle of temperature-time superposition according to which the experimental relaxation functions obtained for different temperatures can be combined with each other using the appropriate coordinate axes stretching.

Materials and methods. Modern technological processes control often requires the modeling of relaxation in reophysically complex media (viscoelastic medium (VEM), capillary-porous bodies (CPB)). Such media are encountered in the production of a wide variety of materials [10-15]. They are extremely important in processes related to oil extraction and transportation [16-18].

Relaxation phenomena in rheophysically complex media are related to the slow development of processes regrouping structural units of different scales. These processes result in deformation changes lag behind the stress change (hysteresis, elastic aftereffect, stress relaxation, etc.) and can be described using the model of elastic bodies with internal friction and viscous bodies with elasticity [10-12,14-15]. Mechanical models of viscoelastic, capillary-porous bodies are helpful for understanding the qualitative characteristics of relaxation phenomena, but their application to the quantitative description of real materials requires the construction of very complex systems consisting of a large number of different springs and viscous elements [19-23].

This research paper shows that the difficulties mentioned hereinabove can be overcome by specifying the structure of time hierarchies which determine the relaxation in rheophysically complex media [24-27]. It is shown that the time fractality allows to simplify the description of relaxation processes resulting in universal relaxation functions of a rather simple form in a wide range of relaxation time [28,29]. It is also shown that in some cases, it is possible to use rheological models with derivatives of fractional order.

Results and discussion. The stress and deformation in viscoelastic and capillary-porous bodies with respect to time. It is necessary to add to the known formula (Eq. 1):

$$\sigma_1 = E \cdot \varepsilon \quad (1)$$

the component taking into account the temporal nature of the stress change (Eq. 2), or (Eq. 3):

$$\sigma_2 = E \cdot \varepsilon \cdot \exp\left(-\frac{t}{\tau}\right), \quad (2)$$

$$\sigma = E \cdot \varepsilon + E \cdot \varepsilon \cdot \exp\left(-\frac{t}{\tau}\right) = E \cdot \varepsilon \left\{1 + \exp\left(-\frac{t}{\tau}\right)\right\}, \quad (3)$$

where σ – body's general stress, E – stress module, ε – elongation ratio (deformation), t – time counted from the moment of the load application, τ – relaxation time.

Elongation ratio (deformation) in the conditions of load application (Eq. 4):

$$\varepsilon = \frac{\sigma}{E \cdot \left\{1 + \exp\left(-\frac{t}{\tau}\right)\right\}}, \quad (4)$$

After unloading, the maximum value of elongation ratio is as follows (Eq. 5):

$$\varepsilon_0 = \frac{\sigma}{E \cdot \left\{1 + \exp\left(-\frac{t}{\tau}\right)\right\}} - \frac{\sigma}{2E} = \frac{\sigma \cdot \left[1 - \exp\left(-\frac{t}{\tau}\right)\right]}{2E \cdot \left[1 + \exp\left(-\frac{t}{\tau}\right)\right]} = \frac{\sigma}{2E} \cdot th\left[+\frac{t}{2\tau}\right], \quad (5)$$

The value of ε_0 allows to determine the current value of its elongation ratio after unloading in time t , i.e., to take into account the unloading aftereffect and thus (Eq. 6):

$$\varepsilon(t) = \varepsilon_0 \cdot \exp\left(-\frac{t}{\tau}\right) = \frac{\sigma}{2E} \cdot th\left[\frac{t}{2\tau}\right] \cdot \exp\left(-\frac{t}{\tau}\right), \quad (6)$$

where t – time running after unloading.

Example 1. The steel wire stretched and rigidly fixed. It is necessary to determine the stress relaxation. Sample data: elastic module $E = 196\,333$ MPa, relaxation time $\tau = 168.2$ s, elongation ratio $\varepsilon = 0.001$ (table 1).

Table 1 – Calculation results of the stresses occurring in the wire and depending on time, i.e., relaxation of stresses

t, s	σ , MPa	t, s	σ , MPa	t, s	σ , MPa
0	392.627	200	256.12	1000	196.66
50	341.89	500	206.26	2000	196.13
100	304.60	900	197.08	3000	196.13

Example 2. The steel sample was stretched to have the stress $\sigma = 300$ MPa, module of elasticity $E = 196\ 333$ MPa, relaxation time $\tau = 168.2$ s, the sample is unloaded at the time $t_p = 1000$ s. It is necessary to determine the value of elongation ratio depending on time. The formula (4) allows to determine the value of elongation ratio in the conditions of loading, the formula (5) allows to calculate the unloading moment. Varying in time residual deformation can be found using the formula (6).

Knowing ε_0 , one can find the residual deformation $\varepsilon_{residual} = \varepsilon_r$ using the ratio (6). If the ratio (Eq. 7):

$$\frac{\varepsilon_r}{\varepsilon_0} = \sigma \tag{7}$$

is specified, the period of time (t^*) can be found, then the residual deformation will be (Eq. 8):

$$\varepsilon_r = \sigma \cdot \varepsilon_0 \text{ (in periods } \tau\text{)}. \tag{8}$$

The results for viscoelastic and capillary-porous bodies are presented in table 2.

Table 2 – Results for viscoelastic and capillary-porous bodies

σ	t^* , s	σ	t^* , s	σ	t^* , s
0.1	2.303τ	0.0001	4.210τ	10^{-8}	18.421τ
0.01	4.605τ	0.00001	11.513τ	10^{-10}	23.026τ
0.001	6.908τ	0.000001	13.816τ	10^{-12}	27.631τ

The energy dissipation process when oscillations are applied. The equity (3) which explains the phenomena occurring in the material during its loading-unloading should be used to describe the process in viscoelastic and capillary-porous bodies. Thus, there is a relation between this dependence and the theory of internal friction given in [30-33].

The effect of time on the results of experiments on the tensile of viscoelastic-type materials and capillary-porous bodies. The changing of the destructive stress in time is expressed by the following general formula (Eq. 9):

$$\sigma_t = \sigma_x + (\sigma_0 - \sigma_x) \cdot \exp\left(-\frac{t}{\tau}\right), \tag{9}$$

where σ_t – ultimate breaking stress at the time t , where σ_x – ultimate breaking stress at the time $t = \infty$, t – load application time, τ – relaxation time.

Relaxation of stresses in viscoelastic and capillary-porous bodies. Generalized Maxwell model. Considering the generalized Maxwell model representing a set of parallel connected chains composed of a series of sequentially connected springs and a viscous element. The rheology of such a body is determined by the known relations (Eqs. 10-11):

$$\sigma = \sum_{n=1}^{\infty} \sigma_n, \tag{10}$$

$$\varepsilon_n = \varepsilon_n^{(1)} + \varepsilon_n^{(2)}, \tag{11}$$

where ε – body deformation, σ – stress, $\sigma_n = E_n \cdot \varepsilon_n^{(1)} = \eta_n \cdot D \cdot \varepsilon_n^{(2)}$ – stress, E_n, η_n – spring stiffness and viscous resistance coefficient of element n , $\varepsilon_n^{(1)}, \varepsilon_n^{(2)}$ – elongation of the spring n and displacement of viscous element n , $D = \frac{d}{dt}$ – operator of differentiation.

It is assumed that the values E_n and τ_n are determined by the scaling laws having the following form (Eq. 12):

$$\tau_n = \tau_0 \cdot n^\nu, \tag{12}$$

After taking the logarithm (12), the following formula can be obtained (Eq. 13):

$$\ln E_n = \ln E_0 - n\lambda, \tag{13}$$

Thus, the time-scale invariance should linearly decrease at the same time as n increases.

If $\frac{t}{\tau_0} \rightarrow \infty (A \rightarrow \infty)$ then asymptotic behavior of this integral is easily determined using Laplace method that leads to an expanded exponential law (Kohlrausch law [29,34]) (Eq. 14), where (Eq. 15):

$$\Phi(t) \approx \exp\left[-(t/\tau)^{1/(\nu+1)}\right], \quad (14)$$

$$\tau = (\tau_0 \cdot \lambda^{-\nu}/\nu) \cdot (1 + \nu^{-1})^{1/(\nu+1)}, \quad (15)$$

Thus, the scale invariance of relaxation processes substantially simplifies their description and allows to use a rather simple universal relaxation functions having the form (14). It should be noted that the relaxation function with an exponent equal to (-1/2) can be obtained using Gauss and Bueche molecular theory of viscoelasticity [11]. However, this theory can explain neither the exponent value deviation (which is often observed in practice) nor the origin of the relaxation functions (14). The scale invariance of the relaxation parameters distribution can serve to explain the principle of temperature-time superposition [11] which can be expressed using the following dependence (Eq. 16):

$$\Phi[k(T)t] = k_1(T) \cdot \Phi_0(t), \quad (16)$$

where T_0 – some characteristic temperature, $\Phi(t)$ and $\Phi_0(t)$ – relaxation functions at temperatures T and T_0 , k , k_1 – coefficients depending on temperature (Eq. 17):

$$\{k(T_0) = k_1(T_0) = 1\}, \quad (17)$$

Figure 1 presents approximation results of this curve, parameters of which τ and β were determined using methods of the sensitivity theory [35,36]. Apparently, the stress relaxation curve is quite well described by the law of Kohlrausch. For this curve, the parameter β is equal to 0.5.

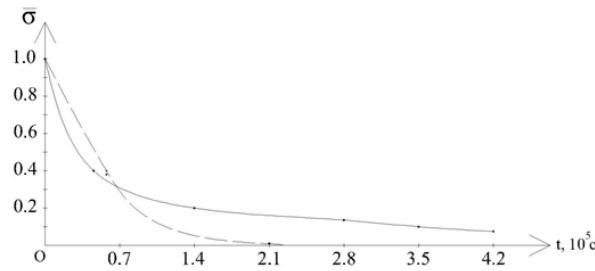


Figure 1 – Relaxation curve approximation

Rheological models of viscoelastic and capillary-porous bodies in fractional derivatives. A viscoelastic/capillary-porous body which can be given using a set of sequentially connected Voigt bodies (chains which consist of parallel connected springs and a viscous element) should be considered now. The stress (Eq. 18) applied to a body at the time $t = 0$.

$$\sigma = \sigma_0 \cdot h(t), \quad (18)$$

Then the deformation rate is determined using the expression (Eq. 19):

$$D\varepsilon(t) = \frac{\sigma_0}{\eta} + \sigma_0 \sum_{n=1}^{\infty} \frac{1}{\eta_n} \cdot \exp(-t/\tau_n), \quad (19)$$

By determining the relaxation function $\Psi(t)$ as (Eq. 20), the following result is obtained (Eq. 21):

$$\Psi = \left\{ D\varepsilon - \frac{\sigma_0}{\eta} \right\} / \sigma_0, \quad (20)$$

$$\Psi(t) = \sum_{n=1}^{\infty} \left(\frac{1}{\eta_n} \right) \cdot \exp(-t/\tau_n), \quad (21)$$

Thus, the time-scale invariance leads to the need to use rheological models in fractional derivatives. It should be noted that such models are entered (on other grounds) in [10,12,35,37].

Relaxation processes in viscoelastic and capillary-porous bodies in conditions of bulk deformation. Considering structural units as viscoelastic elements, the mechanical model given in Figure 2 is proposed to describe the bulk relaxation processes. For this model, the value β_0 characterizes the instantaneous volume compressibility of the medium and the values E_n , η_n ($n = 1, 2, \dots$) describe elasticity of structural units and viscosity forces that counteract their movement.

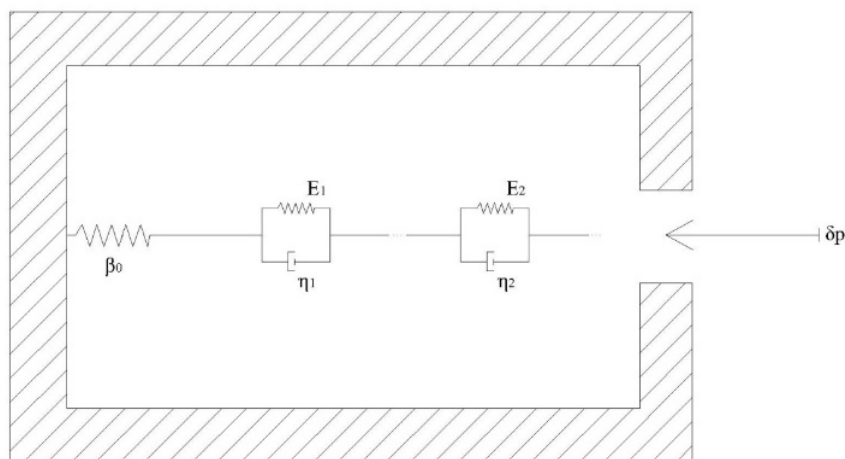


Figure 2 – Mechanical model of volume relaxing body

Similar to the previous cases, it is easy to obtain (Eq. 22):

$$-\delta V(t)/V_0 = \beta_0 \cdot \delta p(t) + \int_0^t \Psi_1(t - \xi) D\delta p(\xi) d\xi, \quad (22)$$

where δV – decrease of medium volume when the pressure is increased by value δp , V_0 – initial volume, $\Psi_1(t) = \beta' \cdot \sum_{n=1}^{\infty} \frac{1}{E_n} \cdot [1 - \exp(-t/\tau_n)]$ – relaxation function, $\tau_n = \eta_n/E_n$, β' – quantity determining the volume changing due to the displacement of the structural elements.

Having differentiated (22) with respect to time, the equation of the viscoelastic medium (capillary-porous body) state is obtained (Eq. 23), where ρ – medium density:

$$\frac{1}{\rho_0} Dp = \beta_0 \cdot Dp + \int_0^t \Psi(t - \xi) Dp(\xi) d\xi, \quad (23)$$

Thus, the pressure curve should be straightened in coordinates (Eq. 24). The inclination of line can be found with respect to χ .

$$Y(S) = \ln \left\{ \frac{1}{S} U - 1 \right\}, \quad \ln S, \quad (24)$$

Equation for relaxing liquid motion. If the motion of a relaxing medium in a pipe (capillary) of radius R is considered then the rheological equation of the medium is presented as follows (Eq. 25):

$$-\frac{\partial v}{\partial r} = \frac{\sigma}{\eta} + \alpha \cdot D^{-\chi} \cdot \frac{\partial \sigma}{\partial t}, \quad (25)$$

where $v(r, t)$ – component of velocity along the pipe axis, σ – shear stress, η – viscosity of medium.

By averaging (25) over the section of the pipe, the following motion equation can be obtained within the frame of the quasi-stationary approximation [38] (Eq. 26):

$$\rho_0 \cdot \left\{ \frac{\partial w}{\partial t} + 2aw \right\} = - \left\{ \frac{\partial p}{\partial x} + \alpha \cdot D^{-\chi} \cdot \frac{\partial^2 p}{\partial x \partial t} \right\}, \quad (26)$$

where w is average cross-sectional velocity, $2a = 8\eta/(\rho_0 R^2)$, $\frac{\partial p}{\partial x}$ is pressure gradient along the axis of the pipe.

As far as is known, the filtration equation can be obtained by throwing away the inertia term and taking $1/2a = k/\eta$ where now w is filtration rate, k is permeability of the porous (capillary-porous) medium; then it is easy to obtain [39-43] (Eq. 27):

$$Dp + \beta_1 \cdot D^{-\chi'} Dp = \kappa (D^{-\chi} \cdot D + 1) \cdot \left(\frac{\partial^2 p}{\partial x^2} \right), \quad (27)$$

where $\kappa = k/\eta^m \cdot \beta_0$ is thermal conduction coefficient, m is porosity.

Algorithm and method for calculating residual deformations in capillary-porous bodies/materials. The following diagram of the capillary-porous body deformation (figure 3) will be considered.

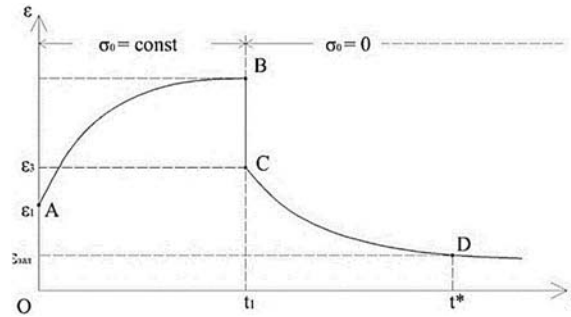


Figure 3 – Diagram of capillary-porous body deformation

The provided below table 3 shows values depending on n .

Table 3 – Values depending on n

N	$\frac{(t_n - t_1)}{\tau}$
10	2.303
10^2	4.605
10^3	6.908
10^9	20.723
10^{10}	23.026

Consequently, CPBs residual deformation is practically reduced to zero (decreasing by 10^{20}) in about 46τ . Thus, if the flow of visitors to the museum (which has pictures modeled as CPB) creates during the visit time $t = t_1$ the load (temperature and humidity) applied on pieces of art and eventually leading to the deformation of these masterpieces then, after visitors leaving the room, their “deformation effect” on pictures practically falls to zero in time interval $t^* \approx t_1 + 46\tau$ where τ is the relaxation time (average) of CPB located in the museum’s premises.

Conclusions. Thus, the distribution of relaxation time in viscoelastic and capillary-porous media may have a scale-invariant (fractal) structure. To confirm this, the spectra of relaxation parameters obtained during experiments carrying out the stretching of polystyrene samples are given in the present paper. It is shown that the indirect confirmation of the scale invariance of relaxation time hierarchy can be the principle of temperature-time superposition.

The present paper shows that in some cases, time fractality can lead to an algebraic relaxation law and, thus, to the need to use rheological models and state equations having fractional derivatives. It is precisely fractional derivatives that can be used for modeling, in particular, bulk relaxation processes. The derived equations of motion of relaxation media in tubes, capillaries, porous media which take into account the time scale invariance of shear and bulk deformation processes are given.

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РЕЛАКСАЦИЯ ҮДЕРІСІНДЕГІ УАҚЫТТЫҚ ИЕРАРХИЯНЫҢ МАСШАБТЫ-ИНВАРИАНТТЫҚ ҚҰРЫЛЫМЫ

Аннотация. Тұтқыр серпімді және қылтүтіктік кеуекті денені жүктеу/жүксіздендіру барысындағы серпімді соңғы әсер құбылысы, кернеуінің бәсеңдеуі кезінде күштік әсер етуде материалдар беталысын ескеретін тербеліс теориясында да ескерілуі тиіс энергияның жинақталуы мен диссипациясы орын алады. Осы-

ларға қатысты серпімді соңғы әсер мен кернеудің бәсеңдеуі қарама-қарсы энергетикалық үдеріс туды-рады, сондықтан да мұндағы негізгі мәселе осындай салдарды түсіну мен заңды анықтау болып саналады.

Зерттеудің мақсаты – тұтқыр серпімді және қылтүктік-кеуекті орталарда релаксация уақытын бөлу масштабты-инварианттық (фракталдық) құрылымға ие бола алатындығын көрсету және түрлі температураға алынған релаксацияның эксперименталды функциясының координата осьтерін сәйкесінше созу арқылы бір-бірін біріктіруге болатын температуралық-уақыттық суперпозиция принципі релаксация уақыты иерархиясының масштабты инварианттылығын жанама түрде растайтындығын көрсету. Тұтқыр серпімділік теориясының әдістері, фракталды талдау және математикалық физика әдістері қолданылды. Мақалада осы екі теория мен академиялық әдебиеттерде сипатталған материалдардың бұзылуына қатысты көптеген эксперименттерді үйлестіру әрекеті жасалды.

Релаксацияның уақыт иерархиясының масштабты инварианттылығын температуралық-уақыттық суперпозиция принципінің жанама түрде растайтындығы сипатталды. Осыған сай түрлі температураға алынған релаксацияның эксперименталды функциялары сәйкесінше координата осьтерін созу арқылы өзара біріктіріле алатындығы көрсетілді.

Түйін сөздер: соңғы әсер, ішкі үйкеліс, тұтқыр серпімділік, механикалық модельдер, ұзару коэффициенті.

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ФРАКТАЛЬНАЯ МАСШТАБНО-ИНВАРИАНТНАЯ СТРУКТУРА ВРЕМЕННОЙ ИЕРАРХИИ В ПРОЦЕССАХ РЕЛАКСАЦИИ

Аннотация. Явление упругого последствия при нагружении/разгрузке вязкоупругих и капиллярно-пористых тел, релаксация их напряжений сопровождается накоплением и диссипацией энергии, которые необходимо учитывать в теории колебаний, которая также учитывает поведение материалов при воздействии силы. Применительно к ним упругое последствие и релаксация напряжений образуют якобы противоположные энергетические процессы, поэтому основная проблема заключается в понимании и открытии законов для таких последствий.

Цель исследования – показать, что распределение времени релаксации в вязкоупругих и капиллярно-пористых средах может иметь масштабно-инвариантную (фрактальную) структуру и что косвенным подтверждением масштабной инвариантности иерархии времен релаксации может быть принцип температур-временная суперпозиция, согласно которой экспериментальные функции релаксации, полученные для различных температур, могут быть объединены друг с другом с помощью соответствующего растяжения осей координат. Использовались методы теории вязкоупругости, фрактального анализа и методы математической физики. Сделана попытка согласовать обе эти теории и многочисленные эксперименты по разрушению материалов, описанные в академической литературе.

Показано, что косвенным подтверждением масштабной инвариантности иерархии времен релаксации может служить принцип температурно-временной суперпозиции, согласно которому экспериментальные функции релаксации, полученные для различных температур, могут быть объединены между собой с помощью соответствующего растяжения осей координат.

Ключевые слова: последствие, внутреннее трение, вязкоупругость, механические модели, коэффициент удлинения.

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