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ИЗВЕСТИЯ

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РЕСПУБЛИКИ КАЗАХСТАН
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SOLITON GEOMETRY USING THE LAX PAIR OF ISOMONODROMIC DEFORMATION

Abstract. Physical processes are described using mathematical models. Many of them are non-linear in nature. For this reason, the theory of a nonlinear medium is relevant and very extensive. From a mathematical point of view, the subject of physics of nonlinear phenomena is systems described by nonlinear partial differential equations that have partial solutions - solitons. A traveling wave that rapidly decreases at infinity is called a solitary wave or soliton. Soliton theory has many fundamental methods for detailed analysis of processes. One of these methods is the geometric interpretation of the physical process.

This paper is devoted to the study of the Lax pair of isomonodromic deformation. The isomonodromy condition is equivalent to the existence of a compatible pair of linear equations, the Lax pair. In this pair, one of the equations undergoes deformation, and the other describes the deformation. Isomonodromic deformation is the theory of isomonodromy (that is, monodromy conservation) of the deformation of ordinary differential equations. This method was used to obtain expressions for the coordinate angle. It is proved that the deformation of a system is isomonodromic if and only if the first and second fundamental forms that define this deformation satisfy the integrability condition. It is shown that, similarly, the area of a soliton surface is represented as a semi-saddle graph.

Key words: Lax pairs, isomonodromic deformation, area, Gaussian curvature,soliton, first and second fundamental forms.

Introduction.

1. The method of isomonodromic deformations appeared in modern science in the works of Jimbo-Miwa-Sato [1-2],[10], and Flaschka-Newel [3], and has been actively used and developed since then. They considered systems of general position, that is, systems in which all irregular singular points are nonresonant. The construction of the surface of isomonodromic deformations became possible in many respects after the proof of several theorems about the properties of systems of linear differential equations. The method of isomonodromic deformations [4],[5] is used to study nonlinear equations, its idea is to implement a nonlinear equation as an isomonodromic condition of a certain system, and this interpretation provides essential information about the nonlinear equation. Under the surface, we consider the set Σ at points $A(x, y, t)$, where x, y, t coordinates are defined in the following

$$x = \varphi(u, v), \quad y = \psi(u, v), \quad z = \chi(u, v). \tag{1}$$

In three-dimensional Euclidean space E^3 surface is represented using the position vector $\vec{r} = \vec{r}(u, v)$ is defined

$$\vec{r} = \vec{r}(u, v) = \varphi(u, v)\vec{i} + \psi(u, v)\vec{j} + \chi(u, v)\vec{k}. \tag{2}$$

where $\vec{i}, \vec{j}, \vec{k}$ are unit vectors directed along the axes of the Cartesian coordinate system. Considering isomonodromic equations, we can describe them as deformations of integrable equations. Lax pair

$$\Phi_x = U\Phi, \qquad \Phi_t = V\Phi, \tag{3}$$

where U, V have form

$$U = \begin{pmatrix} \lambda & -v \\ u & -\lambda \end{pmatrix}, \quad V = \begin{pmatrix} -2\lambda^2 - vu & 2\lambda v + v_x \\ -\lambda u + u_x & 2\lambda^2 + uv \end{pmatrix}. \tag{4}$$

Next, we will deal with three-dimensional space. The standard coordinates of points in this space will be denoted by x, y, t. Our first task is to move from the regions in R^3 to the surfaces in R^3 [6]. To find the inner geometry in a surface [7],[8], we need to find the first fundamental form, i.e., the square of the vector of the differential position vector

$$I = \vec{r}_u^{\,2} du^2 + 2(\vec{r}_u, \vec{r}_v) du dv + \vec{r}_v^{\,2} dv^2. \tag{5}$$

In the surface Σ at each point shows the square shape of the tangent of the differentials du and dv. For the first fundamental form, we use the following notation:

$$E = \vec{r}_u^{\,2}, \quad F = (\vec{r}_u, \vec{r}_v), \quad G = \vec{r}_v^{\,2}. \tag{6}$$

In this case, using (6), we get the following

$$I = E du^2 + 2F du dv + G dv^2. \tag{7}$$

Find the first fundamental form

$$\vec{r}_u^{\,2} = -\frac{1}{2} trace(r_u^2), \quad \vec{r}_{uv} = -\frac{1}{2} trace(\vec{r}_u, \vec{r}_v), \quad \vec{r}_v^{\,2} = -\frac{1}{2} trace(r_v^2), \tag{8}$$

$$\vec{r}_u^{\,2} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad \vec{r}_{uv} = \begin{pmatrix} -4\lambda & 2v \\ 2u & -4\lambda \end{pmatrix}, \quad \vec{r}_v^{\,2} = \begin{pmatrix} 16\lambda^2 - 4uv & 0 \\ 0 & 16\lambda^2 - 4uv \end{pmatrix}, \tag{9}$$

$$E = -1, \quad F = 16\lambda^2 - 4uv, \quad G = 4\lambda. \tag{10}$$

Using (10) we have obtained calculations of the first fundamental form

$$I = -du^2 + \left(-32\lambda^2 + 8uv\right) du dv + +4\lambda dv^2, \tag{11}$$

if

$$d\vec{r} = \vec{r}_u du + \vec{r}_v dv, \qquad \delta\vec{r} = \vec{r}_u \delta u + \vec{r}_v \delta v, \tag{12}$$

then the desired angle will be determined by the formula

$$\cos\omega = \frac{\left(d\vec{r}, \delta\vec{r}\right)}{\sqrt{d\vec{r}^{\,2}}\sqrt{\delta\vec{r}^{\,2}}} \tag{13}$$

$$d\vec{r} = E du^2 + 2F du dv + G dv^2, \qquad \delta\vec{r} = E\delta u^2 + 2F\delta u \delta v + G\delta v^2, \tag{14}$$

$$\left(d\vec{r}, \delta\vec{r}\right) = E du \delta u + F\left(du \delta v + dv \delta u\right) + G dv \delta v, \tag{15}$$

using (14), (15) then the expressions will be as follows

$$\cos\omega = \frac{E du \delta u + F\left(du \delta v + dv \delta u\right) + G dv \delta v}{\sqrt{E du^2 + 2F du dv + G dv^2}\sqrt{E\delta u^2 + 2F\delta u \delta v + G\delta v^2}}, \tag{16}$$

from (16) we can obtain expressions for the coordinate angle

$$du \neq 0, \quad dv = 0, \qquad \delta u = 0, \quad \delta v \neq 0, \tag{17}$$

$$\cos\omega = \frac{F du \delta v}{\sqrt{E du^2}\sqrt{G\delta v^2}} = \frac{F}{\sqrt{EG}} = \frac{-16\lambda^2 + 4uv}{\sqrt{-4\lambda}}, \tag{18}$$

The cross-product, as is known, the module is equal to the area of the parallelogram constructed on the vectors of the cofactors, which in turn is equal to the product of their modules to the sine of the angle between them (taken positively). In this case, we get an expression for the sine of the angle between the coordinate lines at a given point

$$\sin\omega=\sqrt{1-\cos^2\omega}=\sqrt{1-\frac{EG-F^2}{EG}}=\sqrt{\frac{-4\lambda-\left(-16\lambda^2+4uv\right)^2+4uv}{-4\lambda}}. \qquad (19)$$

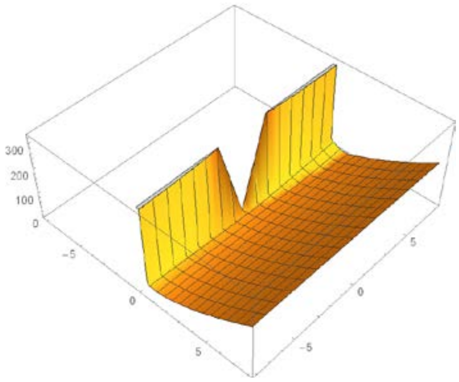

Figure 1 – Sine of the angle between the coordinate lines

2. The second fundamental form

The main issue in the local theory of surfaces is the curvature of the surface, i.e. the nature of its difference from the plane. It turns out that this question is divided into two parts: internal, concerning the geometry of the surface and expressed in terms of its metric, i.e. the first quadratic forms, and external, concerning its location in space, which is another differential form. The shape of shapes in space is a relative concept associated with the choice of a group of transformations of space[9],[10]. As the main group, we take the group of movements of the affine space-transformations that preserve the distance. We have seen that for a curve in space, its shape is completely determined by two functions – the functions of curvature and torsion. For surfaces, a similar fact holds: the shape of a surface is determined by two quadratic shapes. To each point of the surface Σ in addition to the unit normal vector $\vec{n}$

$$n=\frac{\begin{pmatrix}0 & 2v\\ 2u & 0\end{pmatrix}}{4\sqrt{uv}}. \qquad (20)$$

We define the second differential of the position vector

$$d^2r=\vec{r}_{uu}du^2+2\vec{r}_{uv}dudv+\vec{r}_{vv}dv^2+\vec{r}_u d^2u+\vec{r}_v d^2v. \qquad (21)$$

The second fundamental form called the surface $\sum$ is called the scalar product of the vectors d^2r and $\vec{n}$

$$II=\left(d^2\vec{r},\vec{n}\right)=\left(\vec{r}_{uu},\vec{n}\right)du^2+2\left(\vec{r}_{uv},\vec{n}\right)dudv+\left(\vec{r}_{vv},\vec{n}\right)dv^2. \qquad (22)$$

For the coefficients of the second fundamental forms accepted designation

$$e=\left(\vec{r}_{uu},\vec{n}\right),\quad g=\left(\vec{r}_{uv},\vec{n}\right),\quad f=\left(\vec{r}_{vv},\vec{n}\right). \qquad (23)$$

using (23), we can rewrite the second fundamental form so

$$II=edu^2+2\,fdudv+gdv^2. \qquad (24)$$

where

$$e=\frac{2uv}{\sqrt{uv}},\quad f=\frac{8\lambda^2uv+4u^2v^2+4\lambda u_xv-4\lambda uv_x}{\sqrt{uv}},\quad g=\frac{4\lambda uv-u_xv+uv_x}{\sqrt{uv}}. \qquad (25)$$

substituting (25) into (24), we thus got the box of quadratic forms on the surface. Note that its coefficients are the scalar products of the derivatives of the position vector and the normal vector of the surface at a given point

$$II = \frac{2uv}{\sqrt{uv}} du^2 + 2\frac{8\lambda^2 uv + 4u^2v^2 + 4\lambda u_x v - 4\lambda uv_x}{\sqrt{uv}} dudv + \frac{4\lambda uv - u_x v + uv_x}{\sqrt{uv}} dv^2. \tag{26}$$

The second quadratic form provides information about the geometric properties of the surface, primarily about its spatial shape. The physical meaning of the second fundamental form is the shortest distance from another point to the tangent drawn to the first point.

The average curvature is called the half-sum (sometimes the sum)the principal curvatures of H

$$H = \frac{-\left(\left(8\left(\lambda uv + 4\left(-\lambda u - u^2 v\left(4\lambda^2 - uv\right) + u\left(8\lambda^4 - 2\lambda^2 uv\right)\right)\right)\right)\right)}{8\sqrt{uv}\left(\lambda - 4\lambda^2 + uv\right)} + \\ + \frac{v\left(1 + 128\lambda^3 - 32\lambda uv\right)u_x - u\left(1 + 128\lambda^3 - 32uv\right)v_x}{8\sqrt{uv}\left(\lambda - 4\lambda^2 + uv\right)}. \tag{27}$$

Gaussian (or total) curvature (or simply a curvature) is the product of the principal curvatures of K

$$K = \frac{\left(8v\left(-4\lambda^2 uv + u^2 v\left(2\lambda^2 + uv\right)^2\right) + 8\lambda^2 v^2 u_x^2 - u\left(uv + 16\lambda uv\left(2\lambda + uv\right)\right)v}{2uv\left(\lambda + 64\lambda - 32\lambda^2 uv + 4vu^2\right)} + \\ + \frac{8\lambda^2 u^2 v_x^2 + vu_x\left(uv + 16\lambda uv\left(2\lambda^2 + uv\right) - 16\lambda^2 uv_x\right)}{2uv\left(\lambda + 64\lambda - 32\lambda^2 uv + 4vu^2\right)}. \tag{28}$$

here H is the average curvature, K is the Gaussian curvature of a surface at a given point.In addition to lengths and angles, the metric allows you to calculate areas. The tangent vectors that we took to be the sides of the parallelogram at any of the vertices of the subdivision. It can be seen that the sum of these expressions' overall squares is the integral sum for the double integral taken over the domain in which the diffeomorphism of the local map is defined. This fact will be the basis of the definition. So, the area of the area S of the surface is determined by

$$S = \iint EG - F^2 = \iint \sqrt{-4\lambda - \left(-16\lambda^2 + 4uv\right)^2}. \tag{29}$$

All normal sections, excluding the asymptotic one, have a convexity direction in one direction of the normal, but the section of the asymptotic direction may have a complex structure that requires special analysis. In the simplest case, it has an inflection point at a given point, and then the area of the soliton surface has-half-saddle.

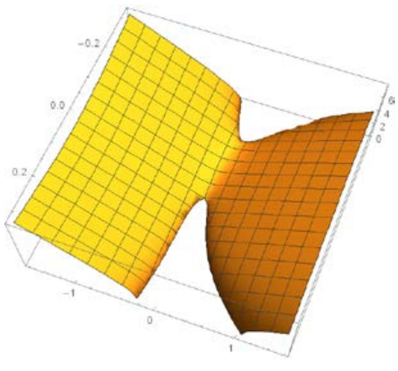

Figure2- Soliton area of isomonodromic deformation

Conclusion

Thus, the problem of isomonodromic deformation using the Lax pair to find the soliton geometry was considered. As a result, the Gaussian curvature and the mean curvature were found in this paper. The most important result here is the first and second fundamental forms. This is the most important object of differential geometry since it defines the internal geometry of the set of solitons, their shape, which does not depend on the location in space. Expressions for the coordinate angle were also found here. And most importantly - the area of the site was found, which has the shape of a surface-a semi-saddle. That is, the properties of soliton solutions of the problem under study were found by this geometric method.

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ИЗОМОНОДРОМДЫ ДЕФОРМАЦИЯНЫҢ ЛАКС ЖҰБЫН ҚОЛДАНУ АРҚЫЛЫ СОЛИТОНДЫҚ ГЕОМЕТРИЯ

Аннотация. Физикалық процестер математикалық модельдердің көмегімен сипатталады. Олардың көпшілігі сызықты емес сипатқа ие. Осы себепті сызықты емес орта теориясы өзекті және өте кең тараған. Математикалық тұрғыдан алғанда, Солитондар сызықтық емес құбылыстар физикасының пәні - жеке шешімдері бар жеке туындылардағы сызықтық емес теңдеулермен сипатталатын жүйелер. Шексіздікке тез түсетін толқын жалғыз толқын немесе солитон деп аталады. Солитон теориясында процестерді егжей-тегжейлі талдаудың көптеген іргелі әдістері бар. Осындай әдістердің бірі-физикалық процесті геометриялық түсіндіру.

Бұл жұмыс изомонодромдық деформацияның лакс жұбын зерттеуге арналған. Изомонодромның жағдайы үйлесімді сызықтық теңдеулер жұбының, Лакс жұбының болуымен тең. Бұл жұпта теңдеулердің бірі деформацияға ұшырайды, ал екіншісі деформацияны сипаттайды. Изомонодромдық деформация-қарапайым дифференциалдық теңдеулердің изомонодромияның (яғни монодромияның сақталуы) деформациясының теориясы. Бұл әдіс координаталық бұрыш үшін өрнектерді алды. Жүйенің деформациясы осы деформацияны анықтайтын бірінші және екінші іргелі формалар интегралдау шартын қанағаттандырған кезде ғана изомонодромды екендігі дәлелденді. Солитон бетінің ауданы жартылай шөгінді график ретінде ұсынылатыны көрсетілген.

Түйін сөздер: Лакс жұптары, изомонодромдық деформация, аудан, Гаусс қисықтығы, солитон, бірінші және екінші іргелі формалар.

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СОЛИТОННАЯ ГЕОМЕТРИЯ С ИСПОЛЬЗОВАНИЕМ ПАРЫ ЛАКСА ИЗОМОНОДРОМНОЙ ДЕФОРМАЦИИ

Аннотация. Физические процессы описываются с помощью математических моделей. Многие из них имеют нелинейный характер. По этой причине теория нелинейной среды актуальна и очень обширна. С математической точки зрения предметом физики нелинейных явлений являются системы, описываемые нелинейными уравнениями в частных производных, которые имеют частные решения – солитоны. Быстроубывающая на бесконечности бегущая волна называется уединенной волной или солитоном. Теория солитонов имеет множество фундаментальных методов для подробного анализа процессов. Одним из таких методов является геометрическая интерпретация физического процесса.

Настоящая работа посвящена изучению пары Лакса изомонодромной деформации. Условие изомонодромии равноценно существованию совместимой пары линейных уравнений, пары Лакса. В этой паре одно из уравнений подвергается деформации, а другое описывает деформацию. Изомонодромная деформация – это теория изомонодромии (то есть сохранения монодромии) деформации обыкновенных дифференциальных уравнений. Этим методом были получены выражения для координатного угла. Доказано, что деформация системы изомонодромна тогда и только тогда, когда первые и вторые фундаментальную формы, задающие данную деформацию, удовлетворяет условию интегрируемости. Показано, что аналогично площадь солитонной поверхность представляется как график- полуседло.

Ключевые слова: пары Лакса, изомонодромная деформация, площадь, гауссова кривизна, солитон, первая и вторая фундаментальные формы.

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REFERENCES

[1] M. Jimbo, T. Miwa, M. Sato.(1979) Holonomic quantum fields II // Publ. RIMS 1979, V. 15. — P. 201-278.

[2] H. Flashka, A. C. Newell. (1980) Monodromy and spectrum preserving deformations // Comm. Math. Phys. — 1980. — V. 76. — P. 67-116.

[3] A. Bolibruch, (1997) On Isomonodromic Deformations of Fuchsian Systems, J. Dynam. Control Systems 3(4) (1997) 589–604.

[4] G. Mahoux, (1999) Introduction to the TheoryofIsomonodromic Deformations of Linear OrdinaryDifferential Equations with Rational Coefficients, Chapter 2 of R. Conte, Ed., The Painlev´e Property: One Century Later, CRM Series in Mathematical Physics. Springer-Verlag (1999).

[5] G. Bauyrzhan, K. Yesmakhanova, K. Yerzhanov, S. Ybyraiymova (2019) Soliton surfaces for complex modified Korteweg-de Vries equation // Conference: 8th International Conference on Mathematical Modeling in Physical Sciences (ICMSQUARE) Location: Bratislava, SLOVAKIA Date: AUG 26-29, 2019

[6]Meirambay, A.; Yerzhanov,(2019) Solution of the deformed Schwarzschild metric by the Yang-Baxter equation. 8TH INTERNATIONAL CONFERENCE ON MATHEMATICAL MODELING IN PHYSICAL SCIENCE. Volume: 1391, number: 012108(2019), Pages: 8-14

[7] A.S. Fokas, M.J. Ablowitz. (1983), On the initial value problem of the second Painlevé Transcendent // Comm. Math. Phys., 91 (1983), pp. 381-403. https://doi.org/10.1007/BF01208781

[8] Flaschka, H., Newell, A.C. (1980), Monodromy- and spectrum-preserving deformations I. // Commun.Math. Phys. 76, 65–116 (1980). https://doi.org/10.1007/BF01197110

[9]M. Jimbo, T. Miwa. (1981), Monodromy preserving deformation of linear ordinary differential equations with rational coefficients. III // Physica D, 4 (1981), pp. 26-46.

[10] Alexander R. ItsVictor Yu. Novoksheno. (1986), The Isomonodromic Deformation Method in the Theory of Painlevé Equations // Springer, Berlin, Heidelberg. 1986

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