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# Х А Б А Р Л А Р Ы

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НАЦИОНАЛЬНОЙ АКАДЕМИИ НАУК  
РЕСПУБЛИКИ КАЗАХСТАН

## NEWS

OF THE NATIONAL ACADEMY OF SCIENCES  
OF THE REPUBLIC OF KAZAKHSTAN

**ФИЗИКА-МАТЕМАТИКА  
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**N E W S**

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**SUBADDITIVITY WEAK MAJORIZATION INEQUALITIES  
FOR  $\tau$ -MEASURABLE OPERATORS**

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**Key words:** subadditivity inequality,  $\tau$ -measurable operator, expansive operator, von Neumann algebra, submajorization.

**Abstract.** Let  $(M, \tau)$  be a semi-finite von Neumann algebra and let  $f : [0, \infty) \rightarrow [0, \infty)$  is increasing concave function such that  $f(0) = 0$ . We have the following results:

- If  $x, y$  are  $\tau$ -measurable operators, then  $f(|x + y|) \preceq f(|x|) + f(|y|)$ .
- If  $x$  is self-adjoint  $\tau$ -measurable operator and  $z \in M$  is expansive operator, then  $f(|z^* x z|) \preceq z^* f(|x|) z$ .

1. **Introduction.** Let  $M_n$  be von Neumann algebra of  $n \times n$  complex matrices, and let  $M_n^+$  be positive part of  $M_n$ . Bourin in [3] proved the following a matrix subadditivity inequality for symmetric norms: Let  $f : [0, \infty) \rightarrow [0, \infty)$  be concave.

- If  $A$  and  $B$  be normal matrices. Then, for all symmetric norms

$$\|f(A + B)\| \leq \|f(A) + f(B)\|.$$

- If  $A$  be normal and let  $Z$  be expansive. Then, for all symmetric norms

$$\|f(Z^* A Z)\| \leq \|Z^* f(A) Z\|.$$

The purpose of this paper is to extend the above results to  $n$ -tuples of  $\tau$ -measurable operators.

This paper is organized as follows. Section 2 contains some preliminary definitions.

In section 3, we proved the weak majorization type of subadditivity inequalities for  $n$  tuples of  $\tau$ -measurable operators.

2. **Preliminaries.** Throughout this paper, we denote by  $M$  a semi-finite von Neumann algebra on the Hilbert space  $H$  with a normalized normal faithful finite trace  $\tau$ . The closed densely defined linear operator  $x$  in  $H$  with domain  $D(x)$  is said to be affiliated with  $M$  if and only if  $u^* x u = x$  for all unitary  $u$  which belong to the commutant  $M'$  of  $M$ . If  $x$  is affiliated with  $M$ , the  $x$  said to be  $\tau$ -measurable if for every  $\varepsilon > 0$  there exists a projection  $e \in M$  such that  $e(H) \subseteq D(x)$  and  $\tau(e^\perp) < \varepsilon$  (where for any projection  $e$  we let  $e^\perp = 1 - e$ ). The set of all  $\tau$ -measure operators will be denoted by  $L_0(M)$ . The set  $L_0(M)$  is a  $*$ -algebra with sum and product being the respective closure of the algebraic sum and product. Let  $P(M)$  be the lattice of projections of  $M$ . The sets

$$N(\varepsilon, \delta) = \{x \in L_0(M) : \exists e \in P(M) \text{ such that } \|xe\| < \varepsilon, \tau(e^\perp) < \delta\}$$

$(\varepsilon, \delta > 0)$  form a base at 0 for an metrizable Hausdorff topology in  $L_0(M)$  called the measure topology. Equipped with the measure topology,  $L_0(M)$  is a complete topological  $*$ -algebra (see [6]). For  $x \in L_0(M)$ , the generalized singular value function  $\mu(x)$  of  $x$  is defined by

$$\mu_s(x) = \inf \{\|xe\|; e \in P(M) \tau(e^\perp) \leq s\}, s \geq 0.$$

If  $x, y \in L_0(M)$ , then we say that  $x$  is submajorized by  $y$  and write  $x \preceq y$  if and only if

$$\int_0^t \mu_s(x) ds \leq \int_0^t \mu_s(y) ds, \quad t \in [0, 1].$$

The following lemma is a well-known result (see [4], Theorem 5.2.).

**Lemma 21.1.** Let  $x, y$  be positive  $\tau$ -measurable operators and  $f : [0, \infty) \rightarrow [0, \infty)$  be a concave function. Then

$$f(x + y) \preceq f(x) + f(y). \tag{1}$$

In particular, Theorem 3.2.8., In [2], we obtain the following result.

**Lemma 2.2.** Let  $f : [0, \infty) \rightarrow [0, \infty)$  be a concave function, and let  $z \in M$  be an expansive operator. Then

$$f(z^* x z) \preceq z^* f(x) z, \quad \forall x \in L_0(M)^+. \tag{2}$$

### 3. Main result

**Theorem 3.12** Let  $x, y$  be  $\tau$ -measurable operators and let and  $f : [0, \infty) \rightarrow [0, \infty)$  is increasing concave function such that  $f(0) = 0$ . Then

$$f(|x + y|) \preceq f(|x|) + f(|y|). \quad (3)$$

*Proof.* Lemma 4.3 in [5] there exist partial isometries  $u, v$  in  $M$  such that

$$|x + y| \preceq u|x|u^* + v|y|v^*$$

Applying Lemma 2.5. (iii) in [5].

$$\mu_s(|x + y|) \leq \mu_s(u|x|u^* + v|y|v^*)$$

Since continuous increasing of  $f$ , we have that

$$f(\mu_s(|x + y|)) \leq f(\mu_s(u|x|u^* + v|y|v^*)).$$

By Lemma 2.5. (iv) in [5], we get

$$\mu_s(f(|x + y|)) \leq \mu_s(f(u|x|u^* + v|y|v^*)).$$

Using inequality (1), we obtain that

$$f(|x + y|) \preceq f(u|x|u^* + v|y|v^*) \preceq f(u|x|u^*) + f(v|y|v^*).$$

By Lemma 2.5. (iii) in [5], we take

$$\mu_s(u|x|u^*) \leq \|u\| \|u^*\| \mu_s(|x|) \leq \mu_s(|x|).$$

Since continuous increasing of  $f$

$$f(\mu_s(u|x|u^*)) \leq f(\mu_s(|x|)).$$

By Lemma 2.5. (iv) in [5], we have that

$$\mu_s(f(u|x|u^*)) \leq \mu_s(f(|x|))$$

so

$$f(u|x|u^*) \preceq f(|x|)$$

and we obtain the same inequality as a following:

$$f(v|y|v^*) \preceq f(|y|).$$

Hence

$$f(|x + y|) \preceq f(|x|) + f(|y|).$$

**Corollary 3.1. 3** Let  $x, y$  be normal  $\tau$ -measurable operators and let and  $f : [0, \infty) \rightarrow [0, \infty)$  is increasing concave function such that  $f(0) = 0$ . Then

$$f(|x + y|) \preceq f(|x|) + f(|y|).$$

**Theorem 3.2. 4** Let  $x$  be self-adjoint  $\tau$ -measurable operators and let and  $f : [0, \infty) \rightarrow [0, \infty)$  is increasing concave function such that  $f(0) = 0$ . Then

$$f(|z^*xz|) \preceq z^*f(|x|)z. \quad (4)$$

*Proof.* It is clear that

$$x \preceq |x|.$$

Then by using Proposition 4.5 (iii) [7]

$$z^* x z \leq z^* |x| z.$$

By Lemma 2.5. (ii),(iii) in [5], we take

$$\mu_s(|z^* x z|) = \mu_s(z^* x z) \leq \mu_s(z^* |x| z).$$

Since continuous increasing of  $f$ , we get

$$f(\mu_s(|z^* x z|)) \leq f(\mu_s(z^* |x| z)).$$

Applying Lemma 2.5. (iv) in [5], we have that

$$\mu_s(f(|z^* x z|)) \leq \mu_s(f(z^* |x| z)).$$

By continuously of integral and inequality (2), we obtain

$$f(|z^* x z|) \leq f(z^* |x| z).$$

**Theorem 5 3.3.** Let  $x$  be  $\tau$ -measurable operators and let and  $f : [0, \infty) \rightarrow [0, \infty)$  is increasing concave function such that  $f(0) = 0$ . Then

$$\begin{pmatrix} f(|z^* x z|) & 0 \\ 0 & f(|z^* x^* z|) \end{pmatrix} \leq \begin{pmatrix} z^* f(|x|) z & 0 \\ 0 & z^* f(|x^*|) z \end{pmatrix}. \quad (5)$$

*Proof.* Applying Theorem 3.2 to the Hermitian operators

$$\begin{pmatrix} 0 & x^* \\ x & 0 \end{pmatrix}$$

we obtain

$$f\left(\begin{pmatrix} z & 0 \\ 0 & z \end{pmatrix}^* \begin{pmatrix} 0 & x^* \\ x & 0 \end{pmatrix} \begin{pmatrix} z & 0 \\ 0 & z \end{pmatrix}\right) \leq \begin{pmatrix} z & 0 \\ 0 & z \end{pmatrix}^* f\left(\begin{pmatrix} 0 & x^* \\ x & 0 \end{pmatrix}\right) \begin{pmatrix} z & 0 \\ 0 & z \end{pmatrix}$$

Hence

$$\begin{aligned} & \begin{pmatrix} f(|z^* x z|) & 0 \\ 0 & f(|z^* x^* z|) \end{pmatrix} \\ &= f\left(\begin{pmatrix} |z^* x z| & 0 \\ 0 & |z^* x^* z| \end{pmatrix}\right) = f\left(\begin{pmatrix} 0 & z^* x^* z \\ z^* x z & 0 \end{pmatrix}\right) \\ &= f\left(\begin{pmatrix} z & 0 \\ 0 & z \end{pmatrix}^* \begin{pmatrix} 0 & x^* \\ x & 0 \end{pmatrix} \begin{pmatrix} z & 0 \\ 0 & z \end{pmatrix}\right) \\ &\leq \begin{pmatrix} z & 0 \\ 0 & z \end{pmatrix}^* f\left(\begin{pmatrix} 0 & x^* \\ x & 0 \end{pmatrix}\right) \begin{pmatrix} z & 0 \\ 0 & z \end{pmatrix} \\ &= \begin{pmatrix} z^* & 0 \\ 0 & z^* \end{pmatrix} f\left(\begin{pmatrix} |x| & 0 \\ 0 & |x^*| \end{pmatrix}\right) \begin{pmatrix} z & 0 \\ 0 & z \end{pmatrix} \\ &= \begin{pmatrix} z^* & 0 \\ 0 & z^* \end{pmatrix} \begin{pmatrix} f(|x|) & 0 \\ 0 & f(|x^*|) \end{pmatrix} \begin{pmatrix} z & 0 \\ 0 & z \end{pmatrix} \\ &= \begin{pmatrix} z^* f(|x|) z & 0 \\ 0 & z^* f(|x^*|) z \end{pmatrix} \end{aligned}$$



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**$\tau$ -ӨЛШЕМДІ ОПЕРАТОРЛАР ҮШІН  
СУБАДДИТИВТІ ӘЛСІЗ МАЖОРЛАНҒАН ТЕҢСІЗДІКТЕР**

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**Тірек сөздер:** субаддитивті теңсіздіктері,  $\tau$ -өлшемді оператор, кеңейтуші оператор, Фон Нейман алгебрасы, субмажорланған.

**Аңдатпа.**  $(M, \tau)$  жартылай ақырлы фон Нейман алгебрасы және  $f: [0, \infty) \rightarrow [0, \infty)$   $f(0)=0$  болатын өспелі ойыс функция болсын. Біз келесі нәтижелерді аламыз.

(1) Егер  $x, y$  тер  $\tau$ -өлшемді операторлар болса, онда

$$f(|x + y|) \leq f(|x|) + f(|y|).$$

(2) Егер  $x$  өзіне-өзі түйіндес  $\tau$ -өлшемді оператор және  $z \in M$  кеңейтуші оператор болса, онда

$$f(|z^* x z|) \leq z^* f(|x|).$$

**СУБАДДИТИВНОСТЬ СЛАБО МАЖОРИЗАЦИОННЫХ НЕРАВЕНСТВ  
ДЛЯ  $\tau$ -ИЗМЕРИМЫХ ОПЕРАТОРОВ**

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**Ключевые слова:** субаддитивное неравенство,  $\tau$ -измеримый оператор, расширяющий оператор, алгебра Фон Неймана, субмажоризация.

**Аннотация.** Пусть  $(M, \tau)$  полу конечная алгебра Фон Неймана и  $f: [0, \infty) \rightarrow [0, \infty)$  возрастающая вогнутая функция с  $f(0)=0$ . Мы получили следующие результаты.

(1) Если  $x, y$   $\tau$ -измеримые операторы, то

$$f(|x + y|) \leq f(|x|) + f(|y|).$$

(2) Если  $x$  самосопряженный  $\tau$ -измеримый оператор и  $z \in M$  расширяющий оператор, то

$$f(|z^* x z|) \leq z^* f(|x|).$$

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