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НАЦИОНАЛЬНОЙ АКАДЕМИИ НАУК  
РЕСПУБЛИКИ КАЗАХСТАН

## NEWS

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OF THE REPUBLIC OF KAZAKHSTAN

**ФИЗИКА-МАТЕМАТИКА  
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## ANALYTICAL SOLUTION OF THE HEAT EQUATION WITH DISCONTINUOUS COEFFICIENTS

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**Key words:** Integral Error Functions, heat polynomials, moving boundaries.

**Abstract.** The main idea of the developed method is based on the use of Integral Error Functions and Heat polynomials which a priori satisfy the heat equation. Linear combination of these functions allows to solve heat equations in domains with moving boundaries with discontinuous coefficients.

**Introduction.** Solution of heat equation by heat polynomials in domains with nonmoving boundaries were considered in [1]. Heat distribution in degenerate domains substantially complicates the problem and cannot be solved by classical methods represented in [2] and [3]. Only for special cases it is possible to construct Heat potentials, however in general solution reduced to the system of Integra-differential equations which contains singularities. In this paper we consider analytical solution of the heat equation with discontinuous coefficients in domain with moving boundary which degenerate at the initial time. This problem is a part of study where the two phase spherical Stefan problem with two moving boundaries has to be solved. We follow the method represented in [4] and utilize Heat polynomials to solve the problem.

**Problem statement.** It is required to find the solution of the Heat Equation

$$\frac{\partial u_1}{\partial t} = a_1^2 \frac{\partial^2 u_1}{\partial x^2}, \quad 0 < x < \beta(t), \quad t > 0 \quad (1),$$

$$\frac{\partial u_2}{\partial t} = a_2^2 \frac{\partial^2 u_2}{\partial x^2}, \quad \beta(t) < x < \infty, \quad t > 0 \quad (2),$$

with following boundary conditions:

$$x = 0: \quad -\lambda_1 \left[ b \frac{\partial u_1}{\partial x} - u_1 \right] \Big|_{x=0} = b^2 P(t), \quad (3),$$

$$u_2(\infty, t) = 0 \quad (4),$$

$$x = \beta(t): \quad u_1(\beta(t), t) = 0 \quad (5),$$

$$u_2(\beta(t), t) = 0 \quad (6).$$

**Problem solution.** We represent solution in the following form:

$$u_1(x, t) = \sum_{n=0}^{\infty} A_{2n} (2a_1 t)^n \left[ i^{2n} \operatorname{erfc} \frac{-x}{2a_1 \sqrt{t}} + i^{2n} \operatorname{erfc} \frac{x}{2a_1 \sqrt{t}} \right] + \sum_{n=0}^{\infty} A_{2n+1} (2a_1 t)^{\frac{2n+1}{2}} \left[ i^{2n+1} \operatorname{erfc} \frac{-x}{2a_1 \sqrt{t}} - i^{2n} \operatorname{erfc} \frac{x}{2a_1 \sqrt{t}} \right] \quad (7),$$

$$u_2(x, t) = \sum_{n=0}^{\infty} C_n (2a_2 t)^{\frac{n}{2}} i^n \operatorname{erfc} \frac{x}{2a_2 \sqrt{t}} \quad (8)$$

Where coefficients  $A_{2n}, A_{2n+1}, C_n$  have to be found. Moreover, it is necessary to find unknown moving  $\beta(t)$ . Using Hermite polynomials we represent (7) in the form of Heat polynomials:

$$u_1(x, t) = \sum_{n=0}^{\infty} A_{2n} \sum_{m=0}^n x^{2n-2m} t^m \beta_{2n,m} + A_{2n+1} \sum_{m=0}^n x^{2n-2m+1} t^m \beta_{2n+1,m} \quad (9)$$

Making substitution  $\sqrt{t} = \tau$

$$u_1(x, t) = \sum_{n=0}^{\infty} A_{2n} \sum_{m=0}^n x^{2n-2m} \tau^{2m} \beta_{2n,m} + A_{2n+1} \sum_{m=0}^n x^{2n-2m+1} \tau^{2m} \beta_{2n+1,m} \quad (10)$$

From (5) for  $x = 0$

$$\left(b \frac{\partial u_1}{\partial x} - u_1\right)_{x=0} = \frac{b^2}{-\lambda_1} P(t)$$

Using above expression we have

$$\begin{aligned} b \sum_{n=0}^{\infty} A_{2n+1} t^n \beta_{2n+1,n} - \sum_{n=0}^{\infty} A_{2n} t^n \beta_{2n,n} &= \frac{b^2}{-\lambda_1} P(t) \\ b A_{2n+1} \beta_{2n+1} - A_{2n} \beta_{2n,n} &= \frac{b^2}{-\lambda_1} \cdot \frac{P^{(n)}(0)}{n!} \\ A_{2n+1} &= A_{2n} \frac{\beta_{2n,n}}{b \cdot \beta_{2n+1,n}} - \frac{b}{\lambda_1 \beta_{2n+1,n}} \cdot \frac{P^{(n)}(0)}{n!} \end{aligned} \quad (11)$$

To find  $A_{2n}$  we use multinomial coefficients of Newton's Polynomials.

It is known that

$$(x_1 + x_2 + \dots + x_m)^n = \sum_{k_1+k_2+\dots+k_m=n} \binom{n}{k_1, k_2, \dots, k_{m+1}} \prod_{1 \leq t \leq m} x_t^{k_t}$$

where  $\binom{n}{k_1, k_2, k_3, \dots, k_m} = \frac{n!}{k_1! k_2! \dots k_m!}$  is a *multinomial coefficient*

$$\text{for } \beta(t) = \alpha_1 t^{1/2} + \alpha_2 t^1 + \alpha_{n+1} t^{3/2} + \dots = \sum_{n=0}^{\infty} \alpha_{n+1} t^{\frac{n+1}{2}} \quad (12)$$

after making substitution  $\tau = \sqrt{t}$  we have

$$(\alpha_1 \tau + \alpha_2 \tau^2 + \dots + \alpha_{m+1} \tau^{m+1})^n = \sum_{k_1+k_2+\dots+k_{m+1}=n} \binom{n}{k_1, k_2, \dots, k_{m+1}} \cdot \alpha_1^{k_1} \alpha_2^{k_2} \dots \alpha_{m+1}^{k_{m+1}} \tau^{k_1+2k_2+\dots+(m+1)k_{m+1}} \quad (13)$$

where

$$\binom{n}{k_1, k_2, \dots, k_{m+1}} \alpha_1^{k_1} \alpha_2^{k_2} \dots \alpha_{m+1}^{k_{m+1}} \quad (14)$$

is a multinomial coefficient in our case

Thus to derive recurrent formula for  $A_{2n}$ , we take both sides of (5),  $2k$  – times derivatives at  $\tau = 0$ , we use multinomial coefficients and get following expressions.

$$0 \equiv 0^{(4l)} = \sum_{n=1}^l A_{2n} \sum_{m=0}^{n-1} C_{2n,m} [4l] + \sum_{n=l+1}^{2l-1} A_{2n} \sum_{m=0}^{2l-n-1} C_{2n,m+2(n-l)} [4l] \beta_{2n,m+2(n-l)} + A_{4l} \beta_{4l,2l} +$$

$$+ \sum_{n=1}^l A_{2n-1} \sum_{m=0}^{n-1} C_{2n-1,m} [4l] \beta_{2n-1,m} + \sum_{n=l+1}^{2l} A_{2n-1} \sum_{m=0}^{2l-n} C_{2n-1,m+2(n-l)-1} [4l] \beta_{2n-1,m+2(n-l)-1}$$

where  $l=1,2,\dots$  and

$$0 \equiv 0^{(2(2l-1))} = \sum_{n=1}^{l-1} A_{2n} \sum_{m=0}^{n-1} C_{2n,m} [2(2l-1)] \beta_{2n,m} + \sum_{n=l}^{2l-2} A_{2n} \sum_{m=1}^{2l-n-1} C_{2n,m+2(n-l)} [2(2l-1)] \beta_{2n,m+2(n-l)} +$$

$$+ A_{4l-2} \beta_{4l-2,2l-1} [2(2l-1)] + \sum_{n=0}^{l-1} A_{2n+1} \sum_{m=0}^n C_{2n+1,m} [2(2l-1)] \beta_{2n+1,m} +$$

$$+ \sum_{n=l}^{2l-2} A_{2n+1} \sum_{m=1}^{2l-n-1} C_{2n+1,m+2(n-l)+1} [2(2l-1)] \beta_{2n+1,m+2(n-l)+1}$$

$A_0=0$

Thus  $A_{2n}$ , coefficients are found explicitly and can be calculated from (15) and (16) where  $C_{i,j}[4l]$  or  $C_{i,j}[4l-2]$  multinomial coefficients or sums of coefficients at  $\beta_{i,j}$ .

To calculate  $C_n$  we apply Leibniz, Faa Di Bruno's formulas and Bell polynomials

Using Leibniz formula we have

$$\frac{\partial^k [2^{n/2} \tau^n \cdot i^n \operatorname{erfc} \beta]}{\partial \tau^k} \Big|_{\tau=0} = \begin{cases} 0, & \text{for } k < n \\ \frac{2^{n/2} k!}{(k-n)!} \cdot [i^n \operatorname{erfc} \beta]^{(k-n)}, & \text{for } k \geq n \end{cases} \quad (17)$$

Using Faa Di Bruno's formula and Bell polynomials for a derivative of a composite function we have

$$\frac{\partial^{k-n} [i^n \operatorname{erfc}(\pm\beta)]}{\partial \tau^{k-n}} \Big|_{\tau=0} = \sum_{m=1}^{k-n} (i^n \operatorname{erfc}(\pm\beta))^{(m)} \Big|_{\beta=0} \cdot B_{k-n,m}(\beta'(\tau), \beta''(\tau), \dots, \beta^{(k-n-m+1)}(\tau)) \Big|_{\tau=0} \quad (18)$$

where

$$B_{k-n,m} = \sum \frac{(k-n)!}{j_1! j_2! \dots j_{k-n-m+1}!} \cdot \beta_1^{j_1} \beta_2^{j_2} \beta_3^{j_3} \dots \beta_{k-n-m+1}^{j_{k-n-m+1}} \quad (19)$$

and  $j_1, j_2, \dots$  satisfy following equations

$$j_1 + j_2 + \dots + j_{k-n-m+1} = m$$

$$j_1 + 2j_2 + \dots + (k-n-m+1)j_{k-n-m+1} = k-n$$

for  $k \geq n$

$$[i^n \operatorname{erfc}(\pm\beta)]^{(m)} \Big|_{\beta=0} = (-1)^m i^{n-m} \operatorname{erfc} 0 = (\mp 1)^m \frac{\Gamma(\frac{n-m+1}{2})}{(n-m)! \sqrt{\pi}} \quad (20)$$

From  $x = \beta(\tau)$  we have

$$\sum_{n=0}^{k-1} C_n \cdot \mu + C_k \cdot \mu = 0 \quad (21)$$

where

$$\mu = (2)^{1/2} \frac{k!}{(k-n)!} \sum_{m=1}^{k-n} (-1)^m \frac{\Gamma(\frac{n-m+1}{2})}{(n-m)! \sqrt{\pi}} \cdot \sum \frac{(k-n)!}{j_1! j_2! \dots j_{k-n-m+1}!} \cdot \beta_1^{j_1} \beta_2^{j_2} \beta_3^{j_3} \dots \beta_{k-n-m+1}^{j_{k-n-m+1}}$$

Thus utilizing (11), (15), (16) and (21) we find  $A_{2n}, A_{2n+1}, C_n$  coefficients of functions (7) and (8). Convergence of (7) and (8) can be proved by following the analogy of the proof represented in [4].

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#### ЖЫЛУӨТКІЗГІШТІК ТЕНДЕУІНІҢ ЖЫЛЖЫМАЛЫ АЙМАҚТАРДА АНАЛИТИКАЛЫҚ ШЕШІМІ

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#### АНАЛИТИЧЕСКОЕ РЕШЕНИЕ УРАВНЕНИЯ ТЕПЛОПРОВОДНОСТИ С РАЗРЫВНЫМИ КОЭФФИЦИЕНТАМИ

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**Ключевые слова:** интегральная Функция Ошибок, тепловые полиномы, подвижные границы.

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