ISSN 2518-1726 (Online),
ISSN 1991-346X (Print)

ҚАЗАҚСТАН РЕСПУБЛИКАСЫ
ҰЛТТЫҚ ҒЫЛЫМ АКАДЕМИЯСЫНЫҢ

# Х А Б А Р Л А Р Ы

# ИЗВЕСТИЯ

НАЦИОНАЛЬНОЙ АКАДЕМИИ НАУК
РЕСПУБЛИКИ КАЗАХСТАН

# N E W S

OF THE NATIONAL ACADEMY OF SCIENCES
OF THE REPUBLIC OF KAZAKHSTAN

## ФИЗИКА-МАТЕМАТИКА СЕРИЯСЫ

♦

## СЕРИЯ ФИЗИКО-МАТЕМАТИЧЕСКАЯ

♦

## PHYSICO-MATHEMATICAL SERIES

**1 (317)**

**ҚАҢТАР – АҚПАН 2018 ж.**
**ЯНВАРЬ – ФЕВРАЛЬ 2018 г.**
**JANUARY – FEBRUARY 2018**

1963 ЖЫЛДЫҢ ҚАҢТАР АЙЫНАН ШЫҒА БАСТАҒАН
ИЗДАЕТСЯ С ЯНВАРЯ 1963 ГОДА
PUBLISHED SINCE JANUARY 1963

ЖЫЛЫНА 6 РЕТ ШЫҒАДЫ
ВЫХОДИТ 6 РАЗ В ГОД
PUBLISHED 6 TIMES A YEAR

АЛМАТЫ, ҚР ҰҒА
АЛМАТЫ, НАН РК
ALMATY, NAS RK

*NAS RK is pleased to announce that News of NAS RK. Series of physico-mathematical scientific journal has been accepted for indexing in the Emerging Sources Citation Index, a new edition of Web of Science. Content in this index is under consideration by Clarivate Analytics to be accepted in the Science Citation Index Expanded, the Social Sciences Citation Index, and the Arts & Humanities Citation Index. The quality and depth of content Web of Science offers to researchers, authors, publishers, and institutions sets it apart from other research databases. The inclusion of News of NAS RK. Series of physico-mathematical in the Emerging Sources Citation Index demonstrates our dedication to providing the most relevant and influential content of physics and mathematics to our community.*

*Қазақстан Республикасы Ұлттық ғылым академиясы "ҚР ҰҒА Хабарлары. Физика-математика сериясы" ғылыми журналының Web of Science-тің жаңаланған нұсқасы Emerging Sources Citation Index-те индекстелуге қабылданғанын хабарлайды. Бұл индекстелу барысында Clarivate Analytics компаниясы журналды одан әрі the Science Citation Index Expanded, the Social Sciences Citation Index және the Arts & Humanities Citation Index-ке қабылдау мәселесін қарастыруда. Webof Science зерттеушілер, авторлар, баспашылар мен мекемелерге контент тереңдігі мен сапасын ұсынады. ҚР ҰҒА Хабарлары. Физика-математика сериясы Emerging Sources Citation Index-ке енуі біздің қоғамдастық үшін ең өзекті және беделді физика-математика бойынша контентке адалдығымызды білдіреді.*

*НАН РК сообщает, что научный журнал «Известия НАН РК. Серия физико-математическая» был принят для индексирования в Emerging Sources Citation Index, обновленной версии Web of Science. Содержание в этом индексировании находится в стадии рассмотрения компанией Clarivate Analytics для дальнейшего принятия журнала в the Science Citation Index Expanded, the Social Sciences Citation Index и the Arts & Humanities Citation Index. Web of Science предлагает качество и глубину контента для исследователей, авторов, издателей и учреждений. Включение Известия НАН РК. Серия физико-математическая в Emerging Sources Citation Index демонстрирует нашу приверженность к наиболее актуальному и влиятельному контенту по физике и математике для нашего сообщества.*

**«ҚР ҰҒА Хабарлары. Физика-математикалық сериясы».**

**ISSN 2518-1726 (Online), ISSN 1991-346X (Print)**

Меншіктенуші: «Қазақстан Республикасының Ұлттық ғылым академиясы» РҚБ (Алматы қ.)
Қазақстан республикасының Мәдениет пен ақпарат министрлігінің Ақпарат және мұрағат комитетінде 01.06.2006 ж. берілген **№5543-Ж** мерзімдік басылым тіркеуіне қойылу туралы куәлік

Мерзімділігі: жылына 6 рет.
Тиражы: 300 дана.

Редакцияның мекенжайы: 050010, Алматы қ., Шевченко көш., 28, 219 бөл., 220, тел.: 272-13-19, 272-13-18, www:nauka-nanrk.kz / physics-mathematics.kz



Типографияның мекенжайы: «Аруна» ЖК, Алматы қ., Муратбаева көш., 75.

**«Известия НАН РК. Серия физико-математическая».**

**ISSN 2518-1726 (Online), ISSN 1991-346X (Print)**

Собственник: РОО «Национальная академия наук Республики Казахстан» (г. Алматы)
Свидетельство о постановке на учет периодического печатного издания в Комитете информации и архивов Министерства культуры и информации Республики Казахстан **№5543-Ж**, выданное 01.06.2006 г.

Периодичность: 6 раз в год.
Тираж: 300 экземпляров.

Адрес редакции: 050010, г. Алматы, ул. Шевченко, 28, ком. 219, 220, тел.: 272-13-19, 272-13-18,
www:nauka-nanrk.kz / physics-mathematics.kz



Адрес типографии: ИП «Аруна», г. Алматы, ул. Муратбаева, 75.

**News of the National Academy of Sciences of the Republic of Kazakhstan. Physical-mathematical series.**

**ISSN 2518-1726 (Online), ISSN 1991-346X (Print)**

Owner: RPA "National Academy of Sciences of the Republic of Kazakhstan" (Almaty)
The certificate of registration of a periodic printed publication in the Committee of information and archives of the Ministry of culture and information of the Republic of Kazakhstan N 5543-Ж, issued 01.06.2006

Periodicity: 6 times a year
Circulation: 300 copies

Editorial address: 28, Shevchenko str., of. 219, 220, Almaty, 050010, tel. 272-13-19, 272-13-18,
www:nauka-nanrk.kz / physics-mathematics.kz



Address of printing house: ST "Aruna", 75, Muratbayev str, Almaty

**N E W S**
OF THE NATIONAL ACADEMY OF SCIENCES OF THE REPUBLIC OF KAZAKHSTAN
**PHYSICO-MATHEMATICAL SERIES**
ISSN 1991-346X
Volume 1, Number 317 (2018), 18 – 24

UDC 510.67

**S.S. Baizhanov[1], B.Sh. Kulpeshov[2]**

[1]Institute of Mathematics and Mathematical Modelling, Almaty, Kazakhstan, e-mail: sayan-5225@mail.ru;
[2]International Information Technology University, Almaty, Kazakhstan, e-mail: b.kulpeshov@iitu.kz

# ON EXPANDING COUNTABLY CATEGORICAL WEAKLY O-MINIMAL THEORIES BY BINARY PREDICATES

**Abstract.** In the present work, questions of preservation of model-theoretical properties at expanding a model of a 1-indiscernible countably categorical weakly o-minimal theory by an arbitrary binary predicate are studied. Questions of preservation of model-theoretical properties at expanding of countably categorical weakly o-minimal theories by unary predicates had been before studied. Here thenotionofanequivalence-genera table formula hasbeenintroduced: if $R(x,y)$ is a $p$-stable formula for some non-algebraic 1-type $p$, then $R(x,y)$ is called an equivalence-generatable formula if every $p$-stable convex to the right or convex to the left formula formed from maximal convex subsets of the set $R(M,a)$ for some element $a \in p(M)$ is equivalence-generating. In terms of the introduced notion of an equivalence–generatable formula, a criterion for preserving the countable categoricity of a 1-indiscernible weakly o-minimal expansion by a binary predicate of 1-indiscernible countably categorical weakly o-minimal structures having the convexity rank 1has been obtained.

**Keywords:** weak o-minimality, countable categoricity, 1-indiscernibility, expansion of models, equivalence-generating formula, equivalence relation.

Let $L$ be a countable first-order language. Throughout this paper we consider $L$-structuresandsupposethat $L$ contains the binary relation symbol $<$, which is interpreted as a linear order in these structures. The present paper deals with the concept of *weak o-minimality*, originally deeply studied by D. Macpherson, D. Marker, and C. Steinhorn in [1]. A subset $A$ ofalinearlyorderedstructure $M$ is called *convex*if for any $a,b \in A$ and $c \in M$ whenever $a < c < b$ we have $c \in A$. A *weakly o-minimal* structure is a linearly ordered structure $M = \langle M,=,<,\ldots\rangle$ such that any definable(with parameters) subset of the structure $M$ is a union of finitely many convex sets in $M$. Recall that such a structure $M$ is called *o-minimal* if every definable (with parameters) subset of the structure $M$ is a union of a finite number of intervals and points in $M$. Thus, weak o-minimality is a generalization of o-minimality. Real closed fields with a proper convex valuation ring provide an important example of weakly o-minimal (not o-minimal) structures.

Let $A, B$ be arbitrary subsets of a linearly ordered structure $M$. Then the expression $A < B$ means that $a < b$ whenever $a \in A$ and $b \in B$. The expression $A < b$ means that $A < \{b\}$. We denote by $A^+$ (and, respectively, $A^-$) the set of elements $b$ of the considered structure $M$ with the condition $A < b$ ($b < A$).

Definition 1[2] Let $T$ beaweaklyo-minimaltheory, $M$ be a sufficiently saturated model of the theory $T$, and let $\phi(x)$ be an arbitrary $M$-definable formula with one free variable.

*The convexity rank of the formula* $\phi(x)$ $(RC(\phi(x)))$ is defined as follows:

1) $RC(\phi(x)) \ge 1$ if $\phi(M)$ infinite.

2) $RC(\phi(x)) \ge \alpha + 1$ if there exist a parametrically definable equivalence relation $E(x,y)$ and an infinite number of elements $b_i, i \in \omega$, such that:

• For any $i, j \in \omega$, whenever $i \neq j$ we have $M \vDash \neg E(b_i, b_j)$

• For each $i \in \omega$ $RC(E(x, b_i)) \geq \alpha$ and $E(M, b_i)$ is a convex subset of a set $\phi(M)$

3) $RC(\phi(x)) \geq \delta$, if $RC(\phi(x)) \geq \alpha$ for all $\alpha \leq \delta$ ($\delta$ is limit).

If $RC(\phi(x)) = \alpha$ for some $\alpha$, we say that $RC(\phi(x))$ is defined. Otherwise (i.e. if $RC(\phi(x)) \geq \alpha$ for all $\alpha$), we put $RC(\phi(x)) = \infty$.

Particularly, a theory has convexity rank 1 if there is no definable (with parameters) equivalence relation with an infinite number of convex infinite classes.

In this paper, we investigate the problem of preserving properties for expansions of models of countably categorical weakly o-minimal theories by binary predicates. Earlier in the works [3] – [5], we have studied the problem of preserving properties for expansions of models of countably categorical weakly o-minimal theories by unary predicates. As is known, in the work [6] Baizhanov B.S. proved that an expansion of a model of a weakly o-minimal theory by a unary predicate that distinguishes a finite number of convex sets preserves weak o-minimality of the expanded theory. However, in the case of expanding a model of a weakly o-minimal theory by a binary predicate that distinguishes a finite number of convex sets for each fixed either the first or the second parameter, the expanded theory can lose weak o-minimality (Example 4).

Recall some concepts originally introduced in [1].

Let $Y \subset M^{n+1}$ be an $\varnothing$–definable set, let $\pi : M^{n+1} \to M^n$ be a projection that drops the last coordinate, and let $Z := \pi(Y)$. For each $\bar{a} \in Z$ let $Y_{\bar{a}} := \{y : (\bar{a}, y) \in Y\}$. Suppose that for each $\bar{a} \in Z$ the set $Y_{\bar{a}}$ is bounded above, but has no supremum in $M$. Let $\sim$ — be an $\varnothing$–definable equivalence relation on $M^n$, defined as follows:

$$\bar{a} \sim \bar{b} \text{ for all } \bar{a}, \bar{b} \in M^n \setminus Z, \text{ and } \bar{a} \sim \bar{b} \Leftrightarrow \sup Y_{\bar{a}} = \sup Y_{\bar{b}}, \text{ if } \bar{a}, \bar{b} \in Z.$$

Let $\bar{Z} := Z/\sim$, and for each tuple $\bar{a} \in Z$ we denote by $[\bar{a}]$ $\sim$-class of the tuple $\bar{a}$. There exists a natural $\varnothing$–definable linear order on $M \cup \bar{Z}$, defined as follows. Let $\bar{a} \in Z$ and $c \in M$. Then $[\bar{a}] < c$ if and only if $w < c$ for all $w \in Y_{\bar{a}}$. If it is not true that $\bar{a} \sim \bar{b}$, then there exists a certain $x \in M$ such that $[\bar{a}] < x < [\bar{b}]$ or $[\bar{b}] < x < [\bar{a}]$, and therefore $<$ induces a linear order on $M \cup \bar{Z}$. We call such a set $\bar{Z}$ by *sort* (in this case, $\varnothing$–definable sort) in $\bar{M}$, where $\bar{M}$ is the Dedekind completion of the structure $M$, and consider $\bar{Z}$ as naturally embedded in $\bar{M}$. Similarly, we can obtain sort in $\bar{M}$, considering infima instead of suprema.

Definition 2[1] Let $M$ be a linearly ordered structure, $D \subseteq M$ be an infinite set, $K \subseteq \bar{M}$, $f : D \to K$ be a function. We say that $f$ is *locally increasing (locally decreasing, locally constant)* on $D$ if for any $x \in D$ there exists an infinite interval $J \subseteq D$, containing $x$, so that $f$ is strictly increasing (strictly decreasing, constant) on $J$.

We also say that a function $f$ is *locally monotone* on the set $D \subseteq M$ if $f$ is either locally increasing or locally decreasing on $D$.

Proposition 3[7] Let $M$ be a weakly o-minimal structure, $A \subseteq M$, $p \in S_1(A)$ be a non-algebraic type. Then any function in an $A$-definable sort whose domain contains $p(M)$ is locally monotone or locally constant on $p(M)$.

Example 4 Let $M := \langle \mathrm{R}, < \rangle$ be a linearly ordered structure on the set of real numbers $\mathrm{R}$. It is obvious that $M$ is a model of a countably categorical o-minimal theory. Expand the model $M$ by a new binary relation $S(x, y)$ as follows: let $M' := \langle \mathrm{R}, <, S^2 \rangle$ be such that $S(x, y)$ is the graph of the following unary function $f$, defined as $f(b) = 2b$ for each $b \in \mathrm{Q}$ and $f(c) = -c$ for each $c \in \mathrm{R} \setminus \mathrm{Q}$. It is obvious that for each $a \in M$ $S(a, M)$ and $S(M, a)$ are singleton sets, i.e. convex sets. Nevertheless, note that $M'$ is not weakly o-minimal, since there is no decomposition of the set $\mathbb{R}$ into a finite number of convex sets, on each of which the definable function $f$ is locally monotone or locally constant.

Example 5Let $M := \langle \mathbb{Q}, < \rangle$ be alinearly ordered structure on the set of rational numbers $\mathbb{Q}$. It is obvious that $M$ is a countably categorical o-minimal structure. Expand the model $M$ by a new binary relation $E(x, y)$ as follows:let $M' := \langle \mathbb{Q}, <, E^2 \rangle$ be such that for any $a, b \in \mathbb{Q}$

$$E(a,b) \Leftrightarrow (2n-1)\sqrt{2} < a, b < (2n+1)\sqrt{2}$$

for some $n \in \mathbb{Z}$.

Then it is not difficult to understand that $E(x, y)$ is an equivalence relation that partitions $\mathbb{Q}$ into infinitely many infinite convex classes, and $E$-classes are ordered by the type $\omega^* + \omega$.

It can be proved that $M'$ is a weakly o-minimal structure, but $Th(M')$ is not countable categorical.

Example 6Let $M := \langle \mathbb{Q} \times \mathbb{Q}, <, E^2 \rangle$ be alinearly ordered structure on the set $\mathbb{Q} \times \mathbb{Q}$, ordered lexicographically.The relation $E(x, y)$ is defined as follows:

forany $a = (m_1, n_1), b = (m_2, n_2) \in Q \times Q$ $E(a,b) \Leftrightarrow m_1 = m_2$.

It is obvious that $E(x, y)$ is an equivalence relation that partitions $\mathbb{Q} \times \mathbb{Q}$ into infinitely many infinite convex classes, and the $E$-classes are ordered by the type $\mathbb{Q}$.

Extend the universe $\mathbb{Q} \times \mathbb{Q}$ of the structure $M$ by adding two elements to each $E$-class, which are the left and the right endpoints of the $E$-class.As a result, we obtain a new structure $M' := \langle M', <, E^2 \rangle$. Consider the reduct of the structure $M'$ to the structure $M'' := \langle M', < \rangle$. It is obvious that $M''$ is a countably categorical o-minimal structure. Its expansion $M' := \langle M', <, E^2 \rangle$ is a countably categorical linearly ordered structure, but $Th(M')$ is not weakly o-minimal.

Definition 7[8] Let $M$ be weakly o-minimal structure, $A \subseteq M$, $M$ be $|A|^+$-saturated, $p \in S_1(A)$ benon-algebraic.

(1) An $A$-definableformula $F(x, y)$ iscalled *$p$-stable*, if there exist $\alpha$, $\gamma_1$, $\gamma_2 \in p(M)$ such that $F(M, \alpha) \setminus \{\alpha\} \neq \varnothing$ and $\gamma_1 < F(M, \alpha) < \gamma_2$.

(2) A $p$-stableformula $F(x, y)$ is called *convextothe right (left)*, if there exists $\alpha \in p(M)$ such that $F(M, \alpha)$ is convex, $\alpha$ istheleft (right) endpoint of the set $F(M, \alpha)$ and $\alpha \in F(M, \alpha)$.

InExample 5 the formula $F(x, y) := y \le x \wedge E(x, y)$ is $p$-stable convex to the right, and the formula $G(x, y) := y \ge x \wedge E(x, y)$ is $p$-stable convex to the left, where $p(x) := \{x = x\} \in S_1(\varnothing)$.

Let $F_1(x, y), F_2(x, y)$ be $p$-stable convex to the right (left) formulas. We say that $F_2(x, y)$ *is greater than* $F_1(x, y)$,if there exists $\alpha \in p(M)$ such that $F_1(M, \alpha) \subset F_2(M, \alpha)$.

Definition 8[9] Wesaythata $p$–stable convex to the right (left) formula $F(x, y)$ is*equivalence-generating*, if for any $\alpha, \beta \in p(M)$ such that $M \vDash F(\beta, \alpha)$, the following holds:

$$M \vDash \forall x \left( x \ge \beta \to \left( F(x, \alpha) \leftrightarrow F(x, \beta) \right) \right) \left( M \vDash \forall x \left( x \le \beta \to \left( F(x, \alpha) \leftrightarrow F(x, \beta) \right) \right) \right)$$

Lemma 9[9]. Let $M$ beaweakly o-minimal structure, $A \subseteq M$, $p \in S_1(A)$ benon-algebraic, $M$ be $|A|^+$–saturated. Supposethat $F(x, y)$ isa $p$-stable convex to the right (left) formula, being an equivalence-generating. Then

1) $G(x, y) := F(y, x)$ is a $p$-stable convex to the right (left) formula, being an equivalence-generating.

2) $E(x, y) := F(x, y) \vee F(y, x)$ isanequivalence relation partitioning $p(M)$ into infinitely many infinite convex classes.

Proposition 10[9]Let $T$ bea countably categorical weakly o-minimal theory, $M \models T$, $A \subseteq M$, $p \in S_1(A)$ benon-algebraic. Thenany $p$-stable convex to the right (left) formula is equivalence-generating.

Example 11Let $M := \langle Q, < \rangle$ bealinearly ordered structure on the set of rational numbers $Q$. It is obviousthat $M$ isacountably categorical1-indiscernibleo-minimal structure. Consider the expansion of the structure $M$ by a new binary relation $R(x, y)$: let $M' := \langle Q, <, R^2 \rangle$ such that for any $a, b \in Q$

$$R(a,b) \Leftrightarrow a \le b < a + \sqrt{2}.$$

It is obvious that $R(a, M')$ and $R(M', a)$ are convex for each $a \in M'$. It can be proved that $M'$ isan 1-indiscernibleweakly o-minimal structure.

The formula $F(x, y) := R(y, x)$ is $p$-stable convex to the right, where $p(x) := \{x = x\} \in S_1(\varnothing)$. It is easy to understand that $F(x, y)$ is not equivalence-generating.

Consider the following formulas:

$$R_2(x, y) := \exists t[R(x,t) \wedge R(t, y)], \quad R_n(x, y) := \exists t[R_{n-1}(x,t) \wedge R(t, y)], n \ge 2$$

For each $a \in M'$ we have

$$R(a, M') \subset R_2(a, M') \subset \ldots \subset R_n(a, M') \subset \ldots,$$

from which we obtain that $Th(M')$ is not countably categorical.

Let $M$ be a weakly o-minimal structure, $A \subseteq M$, $p \in S_1(A)$ be non-algebraic, $R(x, y)$ be an $A$-definable formula that is $p$-stable, i.e. for any $a \in p(M)$ there exist $b_1, b_2 \in p(M)$ such that $b_1 < R(M, a) < b_2$.

By weak o-minimality of $M$ theset $R(M, a)$ is the union of a finite number of convex sets.

It is obvious that each of these sets is $A \cup \{a\}$-definable. There exists a finite number of such definable convex sets that are to the left of the element $a$. Denote them by $R_1^l(x, y), \ldots, R_s^l(x, y)$, we assume that

$$R_s^l(M, a) > R_{s-1}^l(M, a) > \ldots > R_1^l(M, a) \ge a.$$

Similarly, there exists a finite number of definable convex sets that are to the right of the element $a$. Denote them by $R_1^r(x, y), \ldots, R_m^r(x, y)$, we assume that

$$a \le R_1^r(M, a) < R_2^r(M, a) < \ldots < R_m^r(M, a).$$

Perhaps there exists a definable convex set whose interior contains an element $a$. Denote it by $R^c(x, y)$. Thus, if $R^c(M, a) \ne \varnothing$, then there exists $b_1, b_2 \in R^c(M, a)$ such that $b_1 < a < b_2$.

Define the following formulas:

$$F^c(x, y) := y \le x \wedge R^c(x, y)$$

$$G^c(x, y) := y \ge x \wedge R^c(x, y)$$

$$F_i^r(x, y) := y \le x \wedge \forall t\, [R_i^r(t, y) \to x < t], 1 \le i \le m$$

$$F_i^{r*}(x, y) := y \le x \wedge \exists t\, [R_i^r(t, y) \wedge x \le t], 1 \le i \le m$$

$$G_j^l(x,y) := y \geq x \wedge \forall t\ [R_j^l(t,y) \to t < x], 1 \leq i \leq s$$

$$G_j^{l*}(x,y) := y \geq x \wedge \exists t\ [R_j^l(t,y) \wedge t \leq x], 1 \leq i \leq s$$

It is obvious that $F^c(x,y), F_i^r(x,y), F_i^{r*}(x,y), 1 \leq i \leq m$ are $p$-stable convex to the rightand formulas $G^c(x,y), G_j^l(x,y), G_j^{l*}(x,y), 1 \leq j \leq s$, are $p$-stable convex to the left.

We say that the formula $R(x,y)$ is*equivalence-generatable*, if every non-trivial formula in the set $\Delta := \{F^c(x,y),\ F_i^r(x,y),\ F_i^{r*}(x,y),\ G^c(x,y),\ G_j^l(x,y),\ G_j^{l*}(x,y) \mid 1 \leq i \leq m, 1 \leq j \leq s\}$ is equivalence-generating.

Example 12Let $M := \langle \mathbb{Q} \times \mathbb{Q}, < \rangle$ be a linearly ordered structure on the set $\mathbb{Q} \times \mathbb{Q}$, ordered lexicographically. It is obvious that $M$ is a countably categorical o-minimal structure.

We introduce the following two binary formulas $E(x,y)$ and $R_1(x,y)$ on the set $\mathbb{Q} \times \mathbb{Q}$: for any $a = (m_1, n_1), b = (m_2, n_2) \in \mathbb{Q} \times \mathbb{Q}$

$$E(a,b) \Leftrightarrow m_1 = m_2$$

$$R_1(a,b) \Leftrightarrow m_1 = m_2 \wedge n_1 \leq n_2 < n_1 + \sqrt{2}$$

Let $R(x,y) := y \leq x \wedge E(x,y) \wedge \neg R_1(x,y)$ and $M' := \langle \mathbb{Q} \times \mathbb{Q}, <, R^2 \rangle$ is an expansion of the model $M$ by binary predicate $R(x,y)$. It is obvious that for any $a \in M'$ $R(M',a)$ is convex and $a < R(M',a)$.

It can be established that $M'$ is a 1-indiscernible weakly o-minimal structure, but $Th(M')$ is not countably categorical.

Consider the following formulas:

$$F_1(x,y) := y \leq x \wedge \forall t\ [R(t,y) \to x < t]$$

$$F_2(x,y) := y \leq x \wedge \exists t\ [R(t,y) \wedge x \leq t]$$

Formulas $F_1(x,y), F_2(x,y)$ are $p$-stable convex to the right, where $p(x) := \{x = x\} \in S_1(\varnothing)$, here $F_2(x,y)$ is equivalence-generating, and $F_1(x,y)$ is not equivalence-generating. Hence, the predicate $R(x,y)$ is not equivalence-generatable.

Theorem 13 Let $M$ be an 1-indiscernible countably categorical weakly o-minimal structure of convexity rank 1, $M'$ is an 1-indiscernible weakly o-minimal expansion of the structure $M$ by a binary predicate $R(x,y)$.

Then $Th(M')$ is countably categorical if and only if the following conditions are satisfied:

(1) $R(x,y)$ and $L(x,y) := R(y,x)$ are equivalence-generatable;

(2) For every $\varnothing$-definable equivalence relation $E(x,y)$, generated by the predicate $R(x,y)$, the set of $E$-classes is densely ordered.

Proof of Theorem 13. $(\Rightarrow)$ Suppose that $Th(M')$ is countably categorical.Consider the predicate $R(x,y)$. Due to the weak o-minimality of structure $M'$ for any $a \in M'$ $R(M',a)$ and $R(a,M')$ are unions of a finite number of convex sets. By the Proposition 10both formulas $R(x,y)$ and $L(x,y)$ must be equivalence-generatable.

Let $E(x,y)$ be an arbitrary $\varnothing$-definable equivalence relation. By the 1-indiscernibility the set of $E$-classes must be either densely ordered without endpoints, or discretely ordered without endpoints.

Whence by countable categoricity, the set of $E$-classes must be densely ordered.

($\Leftarrow$) Let $R(x,y)$ and $L(x,y)$ be equivalence-generatable formulas.Consider an arbitrary $\varnothing$-definable equivalence relation, generated by the predicate $R(x,y)$. By the hypothesis, the set of $E^*$-classes is densely ordered. By 1-indiscernibility there is neither a leftmost $E^*$-class nor a rightmost $E^*$-class. Also, by 1-indiscernibility, there is no $E^*$-class having at least one endpoint (if every $E^*$-class had at least one endpoint, we would have a contradiction with the weak o-minimality of $M'$).

By weak o-minimality of structure $M'$ for any $a \in M'$ $R(M',a)$ and $R(a,M')$ are unions of a finite number of convex sets. Therefore, there are only finitely many formulas of the form $F^c(x,y)$, $F_i^r(x,y)$, $F_i^{r*}(x,y)$, $G^c(x,y)$, $G_j^l(x,y)$, $G_j^{l*}(x,y)$, $1 \le i \le n_1$, $1 \le j \le n_2$ for some $n_1, n_2 < \omega$. Since by the hypothesis $R(x,y)$, $L(x,y)$ are equivalence-generatable formulas, then each non-trivial formula from the list $\Delta := \{F^c(x,y),\ F_i^r(x,y),\ F_i^{r*}(x,y),\ G^c(x,y), G_j^l(x,y),\ G_j^{l*}(x,y) \mid 1 \le i \le n_1, 1 \le j \le n_2\}$ generates an equivalence relation. Thus, we obtain only a finite number of $\varnothing$-definable equivalence relations generated by the predicate $R(x,y)$.

Let $\{E_1(x,y), E_2(x,y), \ldots, E_n(x,y)\}$ be a complete list of $\varnothing$-definable equivalence relations, generated by the predicate $R(x,y)$. By 1-indiscernibility there is no $i, j$ such that $i \ne j$, $1 \le i, j \le n$ and for some $a \in M'$ $E_i(a,M') \subset E_j(a,M')$, $\sup E_j(a,M') = \sup E_j(a,M')$ or $\inf E_i(a,M') = \inf E_j(a,M')$.

Alsothere donot exist such $i, j \in \{1,\ldots,n\}$ that for some $a \in M'$ $E_i(a,M') \setminus E_j(a,M') \ne \varnothing$ and $E_j(a,M') \setminus E_i(a,M') \ne \varnothing$.

Further, for any $1 \le i, j \le n$ if there exists $a \in M'$ such that $E_i(a,M') \subseteq E_j(a,M')$, then for any $a \in M'$ $E_i(a,M') \subseteq E_j(a,M')$. Thus, there is $1 \le m \le n$ (it is possible the situation when for some $i, j \in \{1,\ldots n\}$ $E_i(a,M') = E_j(a,M')$) and possibly some renumbering of the existing equivalence relations in such a way that for any $a \in M'$ we would have

$$E_1(a,M') \subset E_2(a,M') \subset \ldots \subset E_m(a,M').$$

Since, by the hypothesis, the set of $E$-classes is densely ordered for each $\varnothing$-definable equivalence relation $E(x,y)$, then $E_i$-subclasses of each $E_{i+1}$-class are densely ordered without endpoints, where $0 \le i \le m$ and

$$E_0(x,y) := x = y, \quad E_{m+1}(x,y) := x = x \wedge y = y.$$

Further, it can be established by standard methods that $Th(M')$ admits elimination of quantifiers up to atomic formulas and formulas $E_i(x,y)$, $1 \le i \le m$, whence we obtain that $Th(M')$ is countably categorical.

**REFERENCES**

[1] Macpherson H.D., Marker D. and Steinhorn C. Weakly o-minimal structures and real closed fields, Transactions of The American Mathematical Society, volume 352 (**2000**), pp. 5435–5483.

[2] Kulpeshov B.Sh. Weakly o-minimal structures and some of their properties, The Journal of Symbolic Logic, volume 63 (**1998**), pp. 1511–1528.

[3] Baizhanov S.S., Kulpeshov B.Sh. Invariant propertiesatexpandingmodelsofquiteo-minimaltheories, NewsofNationalAcademyodSceincesoftheRepublicofKazakhstan, series physical-mathematical, No.1 (311), **2017**, pp. 65-71.

[4] Baizhanov S.S., Kulpeshov B.Sh. Expansionofmodelsofquiteo-minimaltheoriesbyunarypredicates, ThesesoftheannualscientificconferenceofInstituteofMathematicsandMathematicalModellingCSMESRK, Almaty, **2017**, pp. 16-18.

[5] Baizhanov S.S., Kulpeshov B.Sh. Expansion of modelsofcountablycategoricalweaklyo-

minimaltheoriesbyunarypredicates, Thesesoftheinternationalconference"Actual Problems of Pure and Applied Mathematics'' devoted to the 100-anniversary of academician A.D. Taimanov, Almaty, **2017**, pp. 13-15.

[6] Baizhanov B.S. Expansion of a model of a weakly o-minimal theory by a family of unary predicates, The Journal of Symbolic Logic, volume 66 (**2001**), pp. 1382–1414.

[7] Kulpeshov B.Sh. Countably categorical quite o-minimal theories, Journal of Mathematical Sciences, volume 188, issue 4 (2013), pp. 387–397.

[8] Baizhanov B.S. One-types in weakly o-minimal theories, Proceedings of Informatics and Control Problems Institute, Almaty, **1996**, pp. 75–88.

[9] Baizhanov B.S., Kulpeshov B.Sh. On behaviour of 2-formulas in weakly o-minimal theories, Mathematical Logic in Asia, Proceedings of the 9th Asian Logic Conference (editors S. Goncharov, R. Downey, H. Ono), Singapore, World Scientific, **2006**, pp. 31–40.

**С.С. Байжанов[1], Б.Ш. Кулпешов [2]**

[1]Математика және математикалық модельдеу институты, Алматы, Қазақстан;
[2]Халықаралық ақпараттық технологиялар университеті, Алматы, Қазақстан

## БИНАРЛЫ ПРЕДИКАТТАРМЕН ЕСЕПТІК-КАТЕГОРИЯЛЫҚ БОСАҢ О-МИНИМАЛДЫҚ ТЕОРИЯЛАР БАЙЫТУ ТУРАЛЫ

**Аннотация.**Осы жұмыста кез келген бинарлы предикатпен 1-анықталмалы есептік-категориялық босаң о-минималды құрылымдар байыту кезінде теориятикалық-модельдік қасиеттерді сақталу сұрақтары зерттеленеді. Осының алдында унарлы предикаттармен есептік-категориялық босаң о-минималды теорияларды байыту кезінде теориятикалық-модельдік қасиеттерді сақталу сұрақтарызерттеленді. Эквиваленттік-қалыптасқан формула түсінігі енгізілді: егер $R(x, y)$ – кейбір алгебралық емес 1-тип $p$ үшін $p$-стабильді формула болса, онда $R(x, y)$ эквиваленттік-қалыптасқан формула деп аталады егер кез келген $p$-стабильді оң жаққа қарай дөңесті немесе сол жаққа қарай дөңесті формуласы $R(M, a)$ жиынтығының максималды дөңес шекарасынан қалыптастырылған кейбір $a \in p(M)$ эквиваленттік-өрнекті болады. Енгізілген эквиваленттік-қалыптасқан формула түсінік терминдермен дөңестік рангісі 1 1-анықталмалы есептік-категориялық босаң о-минималды құрылымдарды 1-анықталмалы босаң о-минималды байытында есептік категориялықты сақтау критерийі алынды.

**Тірек сөздер:** босаң о-минималдық, есептік категориялық, 1-анықталмаушылық, модельдер байыту, эквиваленттік-өрнекті формула, эквиваленттік қатынасы.

**С.С. Байжанов [1], Б.Ш. Кулпешов [2]**

[1]Институт математики и математического моделирования, Алматы;
[2]Международный университет информационных технологий, Алматы

## ОБ ОБОГАЩЕНИИ СЧЕТНО КАТЕГОРИЧНЫХ СЛАБО О-МИНИМАЛЬНЫХ ТЕОРИЙ БИНАРНЫМИ ПРЕДИКАТАМИ

**Аннотация.** В настоящей работе исследуются вопросы сохранения теоретико-модельных свойств при обогащениях 1-неразличимых счетно категоричных слабо о-минимальных структур произвольным бинарным предикатом. Ранее исследовались вопросы сохранения теоретико-модельных свойств при обогащениях счетно категоричных слабо о-минимальных теорий унарными предикатами. Введено понятие эквивалентность-генерируемой формулы: если $R(x, y)$ – $p$-стабильная формула для некоторого неалгебраического 1-типа $p$ ,то $R(x, y)$ называется эквивалентность-генерируемой формулой, если любая $p$-стабильная выпуклая вправо или влево формула, образованная из максимальных выпуклых подмножеств множества $R(M, a)$ для некоторого $a \in p(M)$ является эквивалентность-генерирующей.В терминах вновь введенного понятия эквивалетность-генерируемой формулы получен критерий сохранения счетной категоричности 1-неразличимого слабо о-минимального обогащения бинарным предикатом 1-неразличимых счетно категоричных слабо о-минимальных структур ранга выпуклости 1.

**Ключевые слова:** слабая о-минимальность, счетная категоричность, 1-неразличимость, обогащение моделей, эквивалентность-генерирующая формула, отношение эквивалентности.

**МАЗМҰНЫ**



* * *

**СОДЕРЖАНИЕ**



* * *

**CONTENTS**

**Publication Ethics and Publication Malpractice in the journals of the National Academy of Sciences of the Republic of Kazakhstan**

For information on Ethics in publishing and Ethical guidelines for journal publication see http://www.elsevier.com/publishingethics and http://www.elsevier.com/journal-authors/ethics.

Submission of an article to the National Academy of Sciences of the Republic of Kazakhstan implies that the described work has not been published previously (except in the form of an abstract or as part of a published lecture or academic thesis or as an electronic preprint, see http://www.elsevier.com/postingpolicy), that it is not under consideration for publication elsewhere, that its publication is approved by all authors and tacitly or explicitly by the responsible authorities where the work was carried out, and that, if accepted, it will not be published elsewhere in the same form, in English or in any other language, including electronically without the written consent of the copyright-holder. In particular, translations into English of papers already published in another language are not accepted.

No other forms of scientific misconduct are allowed, such as plagiarism, falsification, fraudulent data, incorrect interpretation of other works, incorrect citations, etc. The National Academy of Sciences of the Republic of Kazakhstan follows the Code of Conduct of the Committee on Publication Ethics (COPE), and follows the COPE Flowcharts for Resolving Cases of Suspected Misconduct (http://publicationethics.org/files/u2/New_Code.pdf). To verify originality, your article may be checked by the Cross Check originality detection service http://www.elsevier.com/editors/plagdetect.

The authors are obliged to participate in peer review process and be ready to provide corrections, clarifications, retractions and apologies when needed. All authors of a paper should have significantly contributed to the research.

The reviewers should provide objective judgments and should point out relevant published works which are not yet cited. Reviewed articles should be treated confidentially. The reviewers will be chosen in such a way that there is no conflict of interests with respect to the research, the authors and/or the research funders.

The editors have complete responsibility and authority to reject or accept a paper, and they will only accept a paper when reasonably certain. They will preserve anonymity of reviewers and promote publication of corrections, clarifications, retractions and apologies when needed. The acceptance of a paper automatically implies the copyright transfer to the National Academy of Sciences of the Republic of Kazakhstan.

The Editorial Board of the National Academy of Sciences of the Republic of Kazakhstan will monitor and safeguard publishing ethics.

Правила оформления статьи для публикации в журнале смотреть на сайтах:

www:nauka-nanrk.kz

http://www.physics-mathematics.kz

**ISSN 2518-1726 (Online), ISSN 1991-346X (Print)**

Редакторы *М. С. Ахметова, Т.А. Апендиев, Д.С. Аленов*
Верстка на компьютере *А.М. Кульгинбаевой*

Подписано в печать 15.02.2018.
Формат 60x881/8. Бумага офсетная. Печать – ризограф.
9 п.л. Тираж 300. Заказ 1.

*Национальная академия наук РК*
*050010, Алматы, ул. Шевченко, 28, т. 272-13-18, 272-13-19*