

ISSN 2518-1726 (Online),  
ISSN 1991-346X (Print)

ҚАЗАҚСТАН РЕСПУБЛИКАСЫ  
ҰЛТТЫҚ ҒЫЛЫМ АКАДЕМИЯСЫНЫҢ

# Х А Б А Р Л А Р Ы

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## ИЗВЕСТИЯ

НАЦИОНАЛЬНОЙ АКАДЕМИИ НАУК  
РЕСПУБЛИКИ КАЗАХСТАН

## NEWS

OF THE NATIONAL ACADEMY OF SCIENCES  
OF THE REPUBLIC OF KAZAKHSTAN

**ФИЗИКА-МАТЕМАТИКА  
СЕРИЯСЫ**



**СЕРИЯ**

**ФИЗИКО-МАТЕМАТИЧЕСКАЯ**



**PHYSICO-MATHEMATICAL  
SERIES**

**1 (317)**

**ҚАҢТАР – АҚПАН 2018 ж.  
ЯНВАРЬ – ФЕВРАЛЬ 2018 г.  
JANUARY – FEBRUARY 2018**

1963 ЖЫЛДЫҢ ҚАҢТАР АЙЫНАН ШЫҒА БАСТАҒАН  
ИЗДАЕТСЯ С ЯНВАРЯ 1963 ГОДА  
PUBLISHED SINCE JANUARY 1963

ЖЫЛЫНА 6 РЕТ ШЫҒАДЫ  
ВЫХОДИТ 6 РАЗ В ГОД  
PUBLISHED 6 TIMES A YEAR

АЛМАТЫ, ҚР ҰҒА  
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«ҚР ҰҒА Хабарлары. Физика-математикалық сериясы».

ISSN 2518-1726 (Online), ISSN 1991-346X (Print)

Меншіктенуші: «Қазақстан Республикасының Ұлттық ғылым академиясы» РҚБ (Алматы қ.)  
Қазақстан республикасының Мәдениет пен ақпарат министрлігінің Ақпарат және мұрағат комитетінде  
01.06.2006 ж. берілген №5543-Ж мерзімдік басылым тіркеуіне қойылу туралы куәлік

Мерзімділігі: жылына 6 рет.  
Тиражы: 300 дана.

Редакцияның мекенжайы: 050010, Алматы қ., Шевченко көш., 28, 219 бөл., 220, тел.: 272-13-19, 272-13-18,  
[www.nauka-nanrk.kz](http://www.nauka-nanrk.kz) / [physics-mathematics.kz](http://physics-mathematics.kz)

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Типографияның мекенжайы: «Аруна» ЖК, Алматы қ., Муратбаева көш., 75.

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**«Известия НАН РК. Серия физико-математическая».**

**ISSN 2518-1726 (Online), ISSN 1991-346X (Print)**

Собственник: РОО «Национальная академия наук Республики Казахстан» (г. Алматы)

Свидетельство о постановке на учет периодического печатного издания в Комитете информации и архивов  
Министерства культуры и информации Республики Казахстан №5543-Ж, выданное 01.06.2006 г.

Периодичность: 6 раз в год.

Тираж: 300 экземпляров.

Адрес редакции: 050010, г. Алматы, ул. Шевченко, 28, ком. 219, 220, тел.: 272-13-19, 272-13-18,  
[www.nauka-nanrk.kz](http://www.nauka-nanrk.kz) / [physics-mathematics.kz](http://physics-mathematics.kz)

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Адрес типографии: ИП «Аруна», г. Алматы, ул. Муратбаева, 75.

## NEWS

OF THE NATIONAL ACADEMY OF SCIENCES OF THE REPUBLIC OF KAZAKHSTAN

## PHYSICO-MATHEMATICAL SERIES

ISSN 1991-346X

Volume 1, Number 317 (2018), 63 – 74

UDC 539.2/.6

**K.B.Jakupov**

Institute of mathematics and mathematical modeling, Almaty, Kazakhstan.

Kazak National University named after Al-Farabi

E-mail: [jakupovKB@mail.ru](mailto:jakupovKB@mail.ru)**NONLINEAR HOOKE LAW IN THE THEORY OF ELASTICITY  
OF INHOMOGENEOUS AND ANISOTROPIC BODIES**

**Abstract.** Directly from the physical connection with the nonlinear Hooke law, the components of the stress tensor of a rigid deformable body and new nonlinear equations of the theory of elasticity with an asymmetric stress tensor are derived, as a special case we obtain equations with the linear Hooke's law. The Lamé hypothesis and Lamé's equations do not have a physical connection with Hooke's law, this is their falsehood. Lamé took as a basis the approximate formula of the incomplete differential and suggested in his hypothesis the proportionality of the stress tensor components to the symmetrical half of the given incomplete differential of displacement, and the antisymmetric half of the differential is discarded, which is the result of the false symmetry of the Lamé stress tensor. The new nonlinear equations are approximated by an explicit scheme, with the use of which the elastic state of a flat bar is numerically calculated with the normal and tangential stresses acting on the upper face. The same scheme is applied to the Lamé equations. The obtained patterns of displacements distribution clearly demonstrate the difference in the solutions of the comparable systems of elasticity equations, as well as the discrepancy between the solution of the Lamé equations for a given state of the deformed body. The falsity of Lamé's equations is confirmed theoretically and physically.

**Keywords:** tensile, tangent, normal, stress, tensor.

**1. Tangential stresses according to the generalized Hooke's law**

Hooke's law is an assertion according to which the deformation arising

in an elastic body, is proportional to the applied force. It was discovered in 1660 by the English scientist Robert Hooke.

It should be kept in mind that Hooke's law is satisfied only for small deformations. If the proportionality limit is exceeded, the relationship between stresses and strains becomes nonlinear. For many media, Hooke's law does not apply even for small deformations. The derivation of dynamic equations with an asymmetric stress tensor according to Hooke's linear law is given in [1]:  $\mathbf{F} = k\mathbf{u}$ ,  $k > 0$ ,  $\mathbf{F}_x = u\mathbf{i}$ ,  $\mathbf{F}_y = v\mathbf{j}$ ,  $\mathbf{F}_z = w\mathbf{k}$ ,  $\mathbf{u} = u\mathbf{i} + v\mathbf{j} + w\mathbf{k}$  – vector of displacement, where  $\mathbf{F} = \mathbf{F}_x + \mathbf{F}_y + \mathbf{F}_z$  – external force that causes displacement.

In inhomogeneous media composed of bodies with various elastic properties or in anisotropic bodies whose properties depend on direction, Hooke's law can be nonlinear

$$\mathbf{F} = k_u u^{m_u} \mathbf{i} + k_v v^{m_v} \mathbf{j} + k_w w^{m_w} \mathbf{k}, \quad k_u > 0, k_v > 0, k_w > 0 \quad (1.1)$$

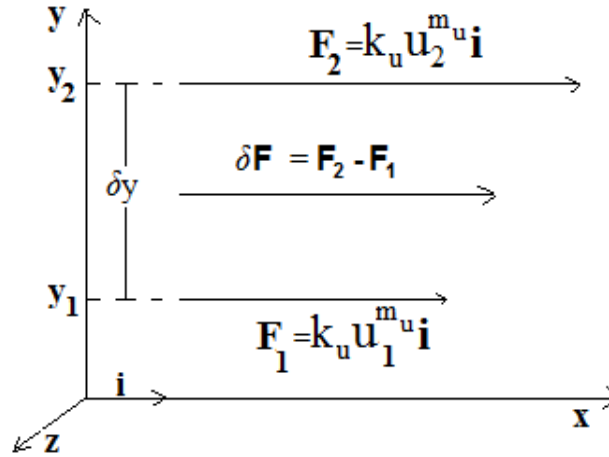
For exponents of 1 and  $k_u = k_v = k_w = k$ , (1.1) becomes Hooke's linear law for an isotropic medium, hence the exponents must be odd numbers [2], which will be confirmed below by the properties of hyperbolic equations of equilibrium. Suppose that on the plane  $y_1$  force is associated with the movement

under the generalized law  $\mathbf{F}_1 = k_u u_1^{m_u} \mathbf{i}$ , similarly force  $\mathbf{F}_2 = k_u u_2^{m_u} \mathbf{i}$  on the surface  $y_2 = y_1 + \delta y$ ,  $\delta y > 0$ .

The increments of forces and displacements between layers are equal:

$$\delta \mathbf{F} = \mathbf{F}_2 - \mathbf{F}_1 = k_u u_2^{m_u} \mathbf{i} - k_u u_1^{m_u} \mathbf{i} = k_u \delta u^{m_u} \mathbf{i}, \quad \delta u^{m_u} = u_2^{m_u} - u_1^{m_u} > 0.$$

Let  $|\mathbf{F}_2| > |\mathbf{F}_1|$ , in this case the force increment is directed along the x axis:  $\delta \mathbf{F} \uparrow \uparrow \mathbf{i}$ .



Linear density is introduced  $\mathbf{f} = \delta \mathbf{F} / \delta y$ ,  $\delta \mathbf{F} = \delta y \mathbf{f}$ . By definition, the average tangent stress vector  $\mathbf{p}_{yxav} = \frac{\delta \mathbf{F}}{\delta x \delta z}$  parallel and equally directed with a force that causes this voltage  $\mathbf{p}_{yxav} \uparrow \uparrow \delta \mathbf{F}$ ,

$$\mathbf{p}_{yxav} \uparrow \uparrow \mathbf{f}.$$

By the introduction of the proportionality coefficient, the following bonds are formed:

$$\mathbf{f} = k' \mathbf{p}_{yxav}, \quad k' > 0, \quad \delta y \mathbf{f} = k' \mathbf{p}_{yxav} \delta y, \quad \mathbf{p}_{yxav} \uparrow \uparrow \mathbf{i},$$

$$k' \mathbf{p}_{yxav} \delta y = k_u \delta u^{m_u} \mathbf{i}$$

This expression is multiplied scalar by the unit vector  $\mathbf{i}$ :

$$(k' \mathbf{p}_{yxav} \delta y, \mathbf{i}) = (k_u \delta u^{m_u} \mathbf{i}, \mathbf{i})$$

As result

$$(k' \mathbf{p}_{yxav} \delta y, \mathbf{i}) = k' |\mathbf{p}_{yxav}| |\delta y| |\mathbf{i}| \cos 0^\circ = k' \mathbf{p}_{yxav} \delta y, \quad (k_u \delta u^{m_u} \mathbf{i}, \mathbf{i}) = k_u \delta u^{m_u}$$

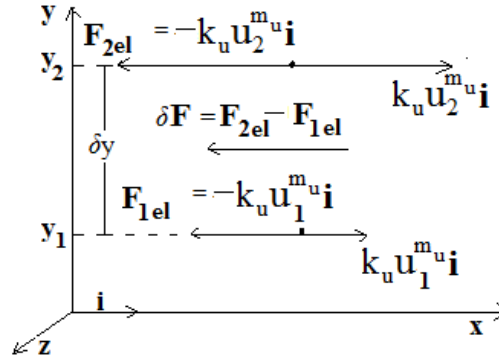
Equalities  $k' \mathbf{p}_{yxav} \delta y = k_u \delta u^{m_u}$ ,  $\mathbf{p}_{yxav} = \frac{k_u}{k'} \frac{\delta u^{m_u}}{\delta y}$  in the limit give a tangential stress

$$\mathbf{p}_{yx} = \lim_{\delta y \rightarrow 0} \frac{k_u}{k'} \frac{\delta u^{m_u}}{\delta y} = \mu_u \frac{\partial u^{m_u}}{\partial y}, \quad \mu_u = \frac{k_u}{k'} > 0$$

Tangential stresses in other directions are obtained analogously:

$$\mathbf{p}_{xy} = \mu_v \frac{\partial v^{m_v}}{\partial x}, \quad \mathbf{p}_{zx} = \mu_u \frac{\partial u^{m_u}}{\partial z}, \quad \mathbf{p}_{xz} = \mu_w \frac{\partial w^{m_w}}{\partial x}, \quad \mathbf{p}_{yz} = \mu_w \frac{\partial w^{m_w}}{\partial y}, \quad \mathbf{p}_{zy} = \mu_v \frac{\partial v^{m_v}}{\partial z}$$

The asymmetric shearing stress formulas are derived for causing stretching of the body of an external force  $\mathbf{F} = \mathbf{F}_x + \mathbf{F}_y + \mathbf{F}_z$ .



The elastic force in the deformed body according to Newton's third law is equal to the external force with a minus sign  $\mathbf{F}_{el} = -\mathbf{F}$ . Consequently, the linear Hooke law for elastic forces will have the form  $\mathbf{F}_{el} = -k_u \mathbf{u}$ ,  $\mathbf{F}_{el} = -k_u i - k_v j - k_w k$ . A similar representation for Hooke's nonlinear law

$$\mathbf{F}_{el} = -k_u u^{m_u} \mathbf{i} - k_v v^{m_v} \mathbf{j} - k_w w^{m_w} \mathbf{k}$$

Suppose that on the plane  $y_1$  the force is connected with the displacement along the nonlinear law  $\mathbf{F}_{1el} = k_u u_1^{m_u} \mathbf{i}$ , similarly acts force  $\mathbf{F}_{2el} = k_u u_2^{m_u} \mathbf{i}$  on surface  $y_2 = y_1 + \delta y$ ,  $\delta y > 0$ .

The increments of forces and displacements between layers are equal:

$$\begin{aligned} \delta \mathbf{F} = \mathbf{F}_{2el} - \mathbf{F}_{1el} &= -k_u u_2^{m_u} \mathbf{i} + k_u u_1^{m_u} \mathbf{i} = -k_u \delta u^{m_u} \mathbf{i}, \\ \delta u^{m_u} &= u_2^{m_u} - u_1^{m_u} > 0. \end{aligned}$$

Let  $|\mathbf{F}_2| > |\mathbf{F}_1|$ , in this case, the force increment is directed against the x axis:  $\delta \mathbf{F} \uparrow \downarrow \mathbf{i}$ . Linear density is introduced  $\mathbf{f} = \delta \mathbf{F} / \delta y$ ,  $\delta \mathbf{F} = \delta y \mathbf{f}$ .

By definition, the average tangential stress vector  $\mathbf{p}_{yxav} = \frac{\delta \mathbf{F}}{\delta \sigma}$ ,  $\delta \sigma = \delta x \delta z$  parallel and equally

directed with a force that causes this voltage  $\mathbf{p}_{yxav} \uparrow \uparrow \delta \mathbf{F}$ ,  $\mathbf{p}_{yxav} \uparrow \uparrow \mathbf{f}$ .

The input of the proportionality coefficient formed a bond:

$$\mathbf{f} = k' \mathbf{p}_{yxav}, k' > 0, \delta y \mathbf{f} = k' \mathbf{p}_{yxav} \delta y, \mathbf{p}_{yxav} \uparrow \downarrow \mathbf{i}, k' \mathbf{p}_{yxav} \delta y = -k_u \delta u^{m_u} \mathbf{i}$$

This expression is multiplied scalar by the unit vector  $\mathbf{i}$ :

$$(k' \mathbf{p}_{yxav} \delta y, \mathbf{i}) = -(k_u \delta u^{m_u} \mathbf{i}, \mathbf{i})$$

As a result

$$(k' \mathbf{p}_{yxav} \delta y, \mathbf{i}) = k' |\mathbf{p}_{yxav}| \delta y |\mathbf{i}| \cos 180^\circ = -k' p_{yxav} \delta y, -(k_u \delta u^{m_u} \mathbf{i}, \mathbf{i}) = -k_u \delta u^{m_u}$$

Equalities

$$k' p_{yxav} \delta y = k_u \delta u^{m_u}, p_{yxav} = \frac{k_u}{k'} \frac{\delta u^{m_u}}{\delta y}$$

in the limit give a tangential stress

$$p_{yx} = \lim_{\delta y \rightarrow 0} \frac{k_u}{k'} \frac{\delta u^{m_u}}{\delta y} = \mu_u \frac{\partial u^{m_u}}{\partial y}, \mu_u = \frac{k_u}{k'} > 0$$

Similarly, tangential stresses are obtained in other directions:

$$p_{xy} = \mu_v \frac{\partial v^{m_v}}{\partial x}, p_{zx} = \mu_u \frac{\partial u^{m_u}}{\partial z}, p_{xz} = \mu_w \frac{\partial w^{m_w}}{\partial x}, p_{yz} = \mu_w \frac{\partial w^{m_w}}{\partial y}, p_{zy} = \mu_v \frac{\partial v^{m_v}}{\partial z}$$

In this way, the stress formulas for external forces coincide with formulas of the action of elastic forces, therefore further conclusions are made only for external forces.

## 2. Relation of Normal Stresses to Hooke's Law

A similar argument establishes the formula of the component  $\mathbf{p}_{xx}^0$  normal voltage  $\mathbf{p}_{xx} = \lambda \operatorname{div} \mathbf{u} \mathbf{i} + \mathbf{p}_{xx}^0$ .

Let the external forces be equal:  $\mathbf{F}_1 = k_u u_1^{m_u} \mathbf{i}$  on surface  $X_1$  and  $\mathbf{F}_2 = k_u u_2^{m_u} \mathbf{i}$  on surface  $X_2 = X_1 + \delta X, \delta X > 0$ .

The increments of forces and displacements between layers are equal:

$$\delta \mathbf{F} = \mathbf{F}_2 - \mathbf{F}_1 = k_u u_2^{m_u} \mathbf{i} - k_u u_1^{m_u} \mathbf{i} = k_u \delta u^{m_u} \mathbf{i}, \delta u^{m_u} = u_2^{m_u} - u_1^{m_u} > 0.$$

By definition, the average stress vector  $\mathbf{p}_{xxav}^0 = \frac{\delta \mathbf{F}}{\delta \sigma}$ ,  $\delta \sigma = \delta y \delta z$  parallel and equally directed with a force that causes this voltage  $\delta \mathbf{F} \uparrow \uparrow \mathbf{i}$ ,  $|\mathbf{F}_2| > |\mathbf{F}_1|$ .

Through the linear density  $\mathbf{f} = \frac{\delta \mathbf{F}}{\delta x}$ ,  $\delta \mathbf{F} = \delta x \mathbf{f}$ ,  $\mathbf{f} = k'' \mathbf{p}_{xxav}^0$  equalities  $\delta \mathbf{F} = k'' \delta x \mathbf{p}_{xxav}^0$ ,  $k'' \delta x \mathbf{p}_{xxav}^0 = k_u \delta u^{m_u} \mathbf{i}$ .

This expression is multiplied scalar by the unit vector  $\mathbf{i}$ :

$$(k'' \delta x \mathbf{p}_{xxav}^0, \mathbf{i}) = (k_u \delta u^{m_u} \mathbf{i}, \mathbf{i})$$

Vectors are parallel in structure  $\mathbf{p}_{xxcp}^0 \uparrow \uparrow \mathbf{i}$ .

Therefore, they occur in scalar products

$$(k'' \delta x \mathbf{p}_{xxav}^0, \mathbf{i}) = k'' \delta x |\mathbf{p}_{xxav}^0| \cdot |\mathbf{i}| \cdot \cos 0^\circ = k'' \delta x p_{xxav}^0, (k_u \delta u^{m_u} \mathbf{i}, \mathbf{i}) = k_u \delta u^{m_u}.$$

The result is  $k'' \delta x p_{xxav}^0 = k_u \delta u^{m_u}$ , whence  $p_{xxav}^0 = \frac{k_u}{k''} \frac{\delta u^{m_u}}{\delta x}$ .

In the limit, the formula for the component of the normal stress

$$p_{xx}^0 = \lim_{\delta x \rightarrow 0} \frac{k_u}{k''} \frac{\delta u^{m_u}}{\delta x} = \mu_u \frac{\partial u^{m_u}}{\partial x}, \mu_u = \frac{k_u}{k''}.$$

The same reasonings also show the components of normal stresses in other directions:

$$p_{ii}^0 = \mu_i \frac{\partial u_i^{m_i}}{\partial x_i}, \mu_i = \frac{k_i}{k''}, i=1,2,3; u_1 \equiv u, u_2 \equiv v, u_3 \equiv w, x_1 \equiv x, x_2 \equiv y, x_3 \equiv z.$$



The same formulas for normal stresses are obtained for the elastic forces in a solid deformed body  $\mathbf{F}_{\text{уп}} = -\mathbf{F}$ .

Thus, the nonlinear Hooke law corresponds to an asymmetric stress tensor in a solid deformed body:

$$p_{ji} = \delta_{ji} \lambda \operatorname{div} \mathbf{u} + \mu_i \varepsilon_{ji}, \quad \varepsilon_{ji} = \frac{\partial u_i}{\partial x_j}, \quad i, j = 1, 2, 3, \quad (2.1)$$

$\mathbf{u} = u_1 \mathbf{i} + u_2 \mathbf{j} + u_3 \mathbf{k}$  - the vector of displacement. In normal stresses  $\lambda \delta_{ji} \operatorname{div} \mathbf{u}$  by Lama is preserved,  $\delta_{ji}$  - the Kronecker symbol.

### 3. On falsifications and inapplicability of Lamé's equations in the nonlinear theory of elasticity of anisotropic bodies

Linear equations of the theory of elasticity of a solid deformable body

$$\rho_0 \frac{\partial^2 \mathbf{u}}{\partial t^2} = \rho_0 \mathbf{F} + (\lambda + \mu) \operatorname{grad} \operatorname{div} \mathbf{u} + \mu \Delta \mathbf{u} \quad (3.1)$$

constructed according to the hypothesis of Lamé on the symmetry of the stress tensor

$$p_{ji} = \lambda \delta_{ji} \operatorname{div} \mathbf{u} + 2\mu \varepsilon_{ji}, \quad \varepsilon_{ji} = \frac{1}{2} \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right), \quad p_{ji} = p_{ij}, \quad i, j = 1, 2, 3, \quad (3.2)$$

$\lambda, \mu$  - the Lamé coefficients.

The Lamé hypothesis is that the elements of the strain tensor  $\varepsilon$  should be equal to the doubled symmetric strain rate tensor, that is, doubled the first half of the artificial formula

$$du_i = \sum_{j=1}^3 \left[ \frac{1}{2} \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) + \left[ \frac{1}{2} \left( \frac{\partial u_i}{\partial x_j} - \frac{\partial u_j}{\partial x_i} \right) \right] dx_j, \quad i = 1, 2, 3, \quad (3.3)$$

(the second antisymmetric half of (3.3) is ignored [3-11]).

The formula (3.3) is formed from the incomplete differential of displacements

$$\tilde{d}u_i = \sum_{j=1}^3 \frac{\partial u_i}{\partial x_j} dx_j, \quad i = 1, 2, 3, \quad (3.4)$$

(Here is the total differential:  $d\mathbf{u} = \frac{\partial \mathbf{u}}{\partial t} dt + \sum_{j=1}^3 \frac{\partial \mathbf{u}}{\partial x_j} dx_j = \frac{\partial \mathbf{u}}{\partial t} dt + \tilde{d}\mathbf{u}$ ).

Thus, in [3-12] the Lamé stress tensor (3.2) does not correspond and does not follow from Hooke's law. Obviously, already by construction, it is not suitable in anisotropic media, especially in the nonlinear Hooke's law.

The above physical method of the author of constructing the stress tensor of an elastic body according to Hooke's law is the opposite of the Lamé hypothesis and exactly follows the definition given by Timoshchenko in [3]: **"The main task of the theory of elasticity is to find, according to the external forces acting on the solid body, those changes in shape that the body undergoes, and those internal elastic forces that, with these changes of form, arise between parts of the body."**

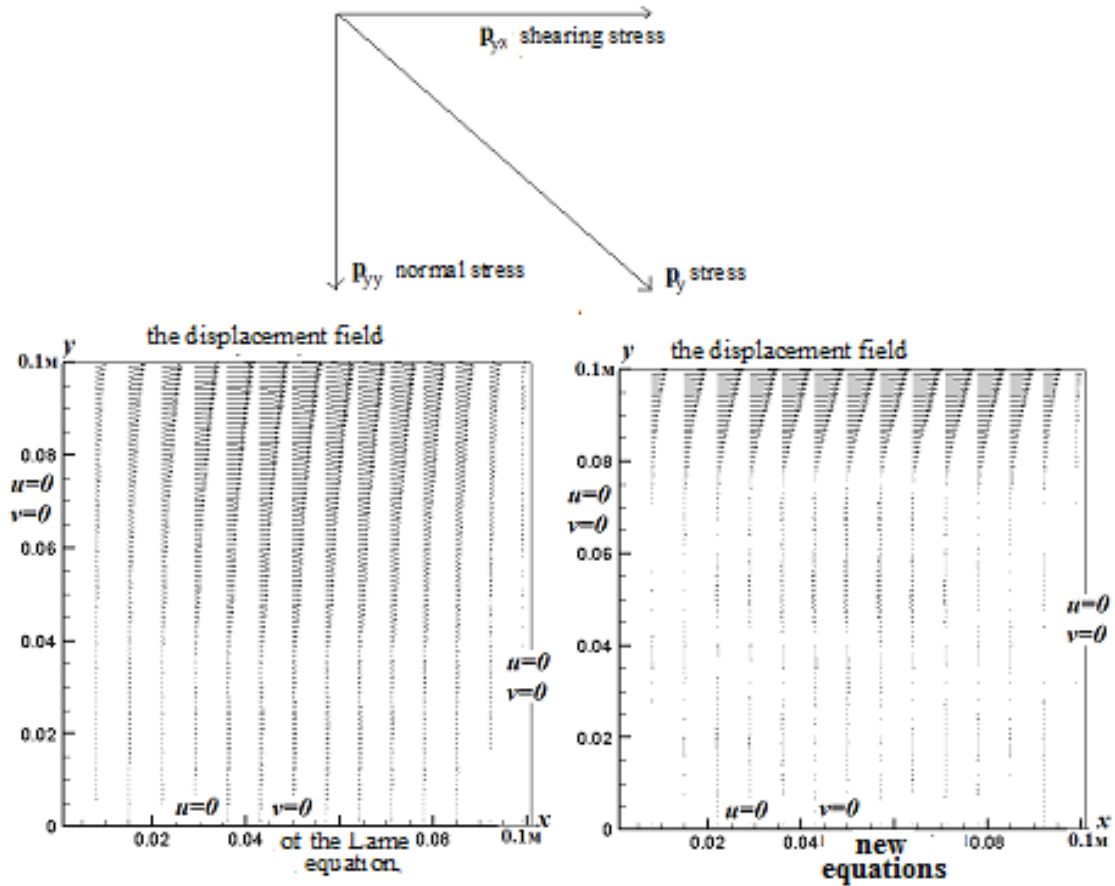


Figure 1

Figure 2

To compare Lamé's equations (3.1) with new equations with asymmetric-a tensor of stresses in an isotropic body  $\mu_u = \mu_v = \mu_w = \mu$ :

$$\rho_0 \frac{\partial^2 \mathbf{u}}{\partial t^2} = \rho_0 \mathbf{F} + \lambda \text{grad div } \mathbf{u} + \mu \Delta \mathbf{u} \quad (3.5)$$

displacements in a square deformable bar measuring 0.1 m by 0.1 m have been calculated.

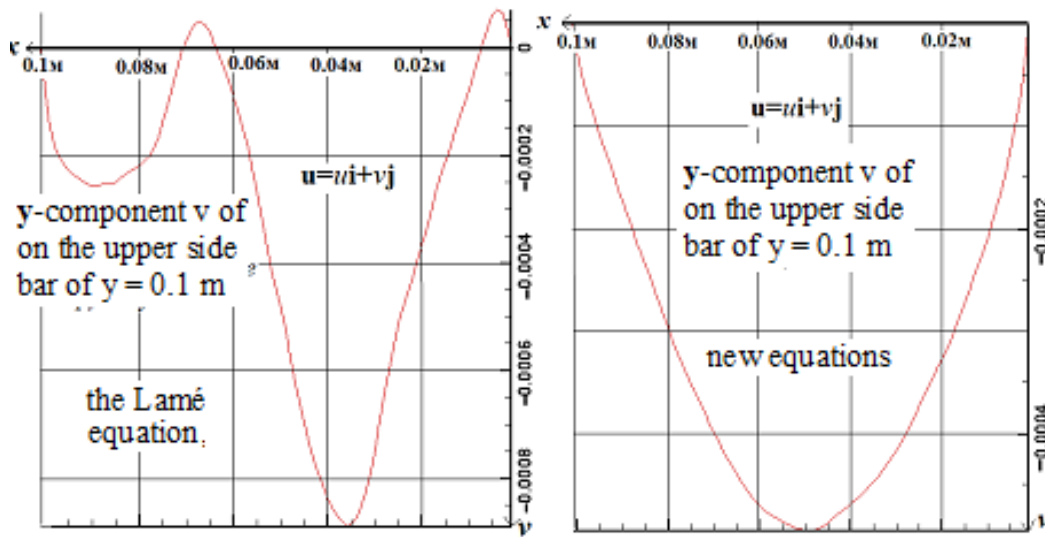


Figure 3

Figure 4

The vector of external force  $\mathbf{p}_y = \mathbf{p}_{yx} + \mathbf{p}_{yy} = p_{yx}\mathbf{i} + p_{yy}\mathbf{j}$  is directed at an angle to the upper plane of the bar. In Fig. 1 and 2 there are the fields of displacement vectors  $\mathbf{u} = u\mathbf{i} + v\mathbf{j}$ , in Fig. 3 and 4 transverse velocity diagrams on the upper side of the bar, all at time  $t = 121.38$  s. Body density  $\rho_0 = 7800 \text{ кг/м}^3$ . Specifically laid  $p_{yy} = -1 \text{ Н/м}^2$ ,  $p_{yx} = 10 \text{ Н/м}^2$ . The other edges of the bar are rigidly fixed, the displacements on them are equal to zero. The Lamé coefficients are chosen equal to  $\lambda = 1 \text{ кг/(с}^2\text{м)}$ ,  $\mu = 100 \text{ кг/(с}^2\text{м)}$ . The two-dimensional Lamé equations (3.1) and the new equations (3.5) are realized by explicit schemes [2] on a  $100 \times 100$  grid with a time step equal to  $0.0005$  s. A clear difference between the numerical solutions, especially the vertical displacements on the upper side of the bar in Fig. 3 and 4.

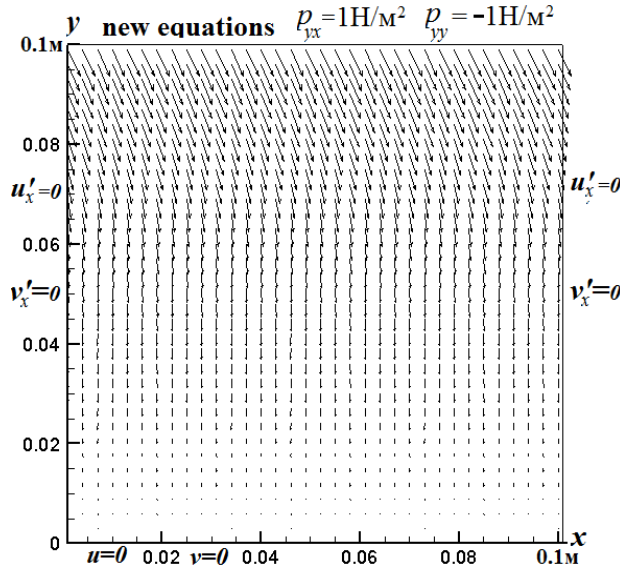


Figura 5

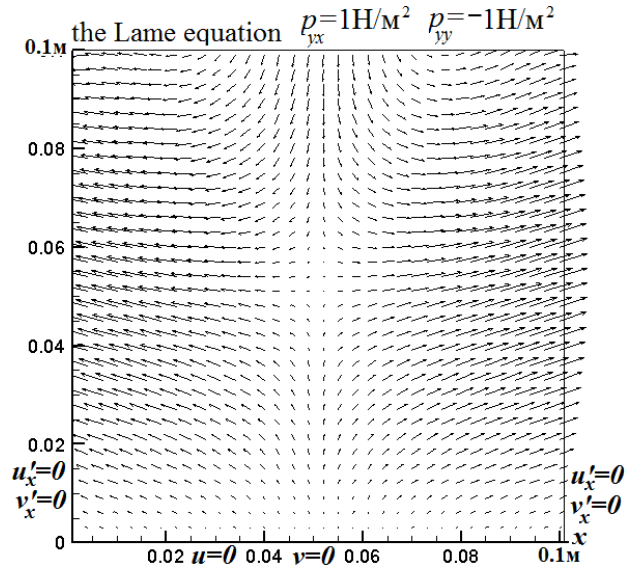


Figura 6

In Fig. 4, the movement of particles of the upper side of the bar  $y = 0.1$  m occurs downward, which is confirmed by negative values of the transverse displacement  $v$  with respect to the new equations. In Fig. 3, Lamé equations have positive values for the transverse component of the displacement, which contradicts the direction of the action of the external force.

In Fig. 5 the displacement field for the new equations; Fig. 6 the field of displacements by the Lamé equations. Stresses act on the entire upper side of the bar. The results of FIG. 1 ± 6 practically confirm the falsity of the Lamé equations with a symmetric stress tensor.

**4. Equations of nonlinear anisotropic theory of elasticity according to the generalized Hooke's law with an asymmetric stress tensor**

Leaving from Hooke's linear law [1], the elements of the strain tensor  $\varepsilon_{ji} = \frac{\partial u_i}{\partial x_j}$ ,  $i, j = 1, 2, 3$  are directly equal to the coefficients of the incomplete differential (3.4).

The asymmetric stress tensor of the generalized Hooke's law

$$p_{ji} = \delta_{ji} \lambda \text{div} \mathbf{u} + \mu_i \varepsilon_{ji}, \quad \varepsilon_{ji} = \frac{\partial u_i}{\partial x_j}, \quad i, j = 1, 2, 3, \tag{4.1}$$

in the equations of the dynamics of a continuous medium in stresses

$$\rho_0 \frac{\partial^2 u_i}{\partial t^2} = \rho_0 F_i + \sum_{j=1}^3 \frac{\partial p_{ji}}{\partial x_j}, \quad i = 1, 2, 3$$

form non-linear equations with exponents in accordance with the generalized Hooke's law

$$\rho_0 \frac{\partial^2 \mathbf{u}_i}{\partial t^2} = \rho_0 \mathbf{F}_i + \lambda \frac{\partial}{\partial x_i} \operatorname{div} \mathbf{u} + \mu_i \Delta \mathbf{u}_i^{m_i}, i=1,2,3 \quad (4.2)$$

### 5. Rationale for the oddness of entire exponents of degree

Equations (4.2) with Cartesian coordinates

$$\rho_0 \frac{\partial^2 \mathbf{u}_i}{\partial t^2} = \rho_0 \mathbf{F}_i + \lambda \frac{\partial}{\partial x_i} \operatorname{div} \mathbf{u} + \mu_i \sum_{j=1}^3 \frac{\partial^2 \mathbf{u}_i^{m_i}}{\partial x_j^2}, i=1,2,3$$

are represented in a differentiated form

$$\rho_0 \frac{\partial^2 \mathbf{u}_i}{\partial t^2} = \rho_0 \mathbf{F}_i + \lambda \frac{\partial}{\partial x_i} \operatorname{div} \mathbf{u} + \mu_i \frac{\partial}{\partial x_j} \left( \frac{\partial \mathbf{u}_i^{m_i}}{\partial x_j} \right), i=1,2,3,$$

where  $\frac{\partial \mathbf{u}_i^{m_i}}{\partial x_j} = m_i \mathbf{u}_i^{m_i-1} \frac{\partial \mathbf{u}_i}{\partial x_j}$ .

Obviously, the equivalent equations

$$\rho_0 \frac{\partial^2 \mathbf{u}_i}{\partial t^2} = \rho_0 \mathbf{F}_i + \lambda \frac{\partial}{\partial x_i} \operatorname{div} \mathbf{u} + \mu_i \frac{\partial}{\partial x_j} \left( m_i \mathbf{u}_i^{m_i-1} \frac{\partial \mathbf{u}_i}{\partial x_j} \right), i=1,2,3 \quad (5.1)$$

are equations of hyperbolic type only for odd exponents of degree  $m_i = 1; 3; 5; 7; 9$  и  $m.d.$ ,  $m_1 \equiv m_u, m_2 \equiv m_v, m_3 \equiv m_w$  for always  $m_i \mathbf{u}_i^{m_i-1} \geq 0$ .

### 6. Explicit diagram of the equations of an anisotropic nonlinear theory of elasticity

The Cauchy-Dirichlet problem for the new equations

$$\begin{aligned} \rho_0 \frac{\partial^2 \mathbf{u}}{\partial t^2} &= \rho_0 \mathbf{F}_x + \lambda \frac{\partial \mathbf{p}}{\partial x} + \mu_u \Delta \mathbf{u}^{m_u}, \\ \rho_0 \frac{\partial^2 \mathbf{v}}{\partial t^2} &= \rho_0 \mathbf{F}_y + \lambda \frac{\partial \mathbf{p}}{\partial y} + \mu_v \Delta \mathbf{v}^{m_v}, \\ \rho_0 \frac{\partial^2 \mathbf{w}}{\partial t^2} &= \rho_0 \mathbf{F}_z + \lambda \frac{\partial \mathbf{p}}{\partial z} + \mu_w \Delta \mathbf{w}^{m_w}, \\ \mathbf{p} &= \frac{\partial \mathbf{u}}{\partial x} + \frac{\partial \mathbf{v}}{\partial y} + \frac{\partial \mathbf{w}}{\partial z} \end{aligned}$$

with the initial conditions:

$$\begin{aligned} \mathbf{u}|_{t=0} &= \mathbf{d}_u(\mathbf{r}), \mathbf{v}|_{t=0} = \mathbf{d}_v(\mathbf{r}), \mathbf{w}|_{t=0} = \mathbf{d}_w(\mathbf{r}), \\ \frac{\partial \mathbf{u}}{\partial t}|_{t=0} &= \mathbf{d}_{uu}(\mathbf{r}), \frac{\partial \mathbf{v}}{\partial t}|_{t=0} = \mathbf{d}_{vv}(\mathbf{r}), \frac{\partial \mathbf{w}}{\partial t}|_{t=0} = \mathbf{d}_{ww}(\mathbf{r}) \end{aligned}$$

and boundary conditions on the boundary  $S$ :

$$\mathbf{u}|_S = \mathbf{q}_u(\mathbf{r}), \mathbf{v}|_S = \mathbf{q}_v(\mathbf{r}), \mathbf{w}|_S = \mathbf{q}_w(\mathbf{r}),$$

In the integration region, a uniform grid is defined  $\overline{\Omega}_h = \{x_i = ih_x, y_j = jh_y, z_k = kh_z, 0 \leq i \leq N_x, 0 \leq j \leq N_y, 0 \leq k \leq N_z\}$ , and a time grid  $\overline{\Omega}_\tau = \{t_n = n\tau, n = 0, 1, \dots, N_\tau\}$ .

Notation of grid functions:  $f_{ijk}^n \equiv f(x_i, y_j, z_k, t_n)$ .

Initial conditions are specified in internal nodes:  $u_{ijk}^0 = d_{uijk}, v_{ijk}^0 = d_{vijk}, w_{ijk}^0 = d_{wijk},$

$$u_{ijk}^1 = d_{uijk} + \tau d_{uuijk}, v_{ijk}^1 = d_{vijk} + \tau d_{vvijk}, w_{ijk}^1 = d_{wijk} + \tau d_{wwijk},$$

$$1 \leq i \leq N_x - 1, 1 \leq j \leq N_y - 1, 1 \leq k \leq N_z - 1$$

Explicit difference scheme:

$$Q_{uijk}^n = \mu_u \left[ \frac{(u_{i-1jk}^n)^{m_u} - 2(u_{ijk}^n)^{m_u} + (u_{i+1jk}^n)^{m_u}}{h_x^2} + \frac{(u_{ij-1k}^n)^{m_u} - 2(u_{ijk}^n)^{m_u} + (u_{ij+1k}^n)^{m_u}}{h_y^2} + \frac{(u_{ijk-1}^n)^{m_u} - 2(u_{ijk}^n)^{m_u} + (u_{ijk+1}^n)^{m_u}}{h_z^2} + \rho_0 F_{xijk} \right],$$

$$Q_{vijk}^n = \mu_v \left[ \frac{(v_{i-1jk}^n)^{m_v} - 2(v_{ijk}^n)^{m_v} + (v_{i+1jk}^n)^{m_v}}{h_x^2} + \frac{(v_{ij-1k}^n)^{m_v} - 2(v_{ijk}^n)^{m_v} + (v_{ij+1k}^n)^{m_v}}{h_y^2} + \frac{(v_{ijk-1}^n)^{m_v} - 2(v_{ijk}^n)^{m_v} + (v_{ijk+1}^n)^{m_v}}{h_z^2} + \rho_0 F_{yijk} \right],$$

$$Q_{wijk}^n = \mu_w \left[ \frac{(w_{i-1jk}^n)^{m_w} - 2(w_{ijk}^n)^{m_w} + (w_{i+1jk}^n)^{m_w}}{h_x^2} + \frac{(w_{ij-1k}^n)^{m_w} - 2(w_{ijk}^n)^{m_w} + (w_{ij+1k}^n)^{m_w}}{h_y^2} + \frac{(w_{ijk-1}^n)^{m_w} - 2(w_{ijk}^n)^{m_w} + (w_{ijk+1}^n)^{m_w}}{h_z^2} + \rho_0 F_{zijk} \right],$$

$$\rho_0 \frac{u_{ijk}^{n+1} - 2u_{ijk}^n + u_{ijk}^{n-1}}{\tau^2} = Q_{uijk}^n + \lambda \left( \frac{u_{i-1jk}^n - 2u_{ijk}^n + u_{i+1jk}^n}{h_x^2} + \right.$$

$$\left. + \frac{v_{i+1j+1k}^n - v_{i+1j-1k}^n - v_{i-1j+1k}^n + v_{i-1j-1k}^n}{4h_x h_y} + \frac{w_{i+1jk+1}^n - w_{i+1jk-1}^n - w_{i-1jk+1}^n + w_{i-1jk-1}^n}{4h_x h_z} \right),$$

$$\rho_0 \frac{v_{ijk}^{n+1} - 2v_{ijk}^n + v_{ijk}^{n-1}}{\tau^2} = Q_{vijk}^n + \lambda \left( \frac{v_{ij-1k}^n - 2v_{ijk}^n + v_{ij+1k}^n}{h_y^2} + \right.$$

$$\left. + \frac{u_{i+1j+1k}^n - u_{i+1j-1k}^n - u_{i-1j+1k}^n + u_{i-1j-1k}^n}{4h_x h_y} + \frac{w_{ij+1k+1}^n - w_{ij+1k-1}^n - w_{ij-1k+1}^n + w_{ij-1k-1}^n}{4h_z h_y} \right),$$

$$\rho_0 \frac{w_{ijk}^{n+1} - 2w_{ijk}^n + w_{ijk}^{n-1}}{\tau^2} = Q_{wijk}^n + \lambda \left( \frac{w_{ijk-1}^n - 2w_{ijk}^n + w_{ijk+1}^n}{h_z^2} + \right.$$

$$+ \frac{u_{i+1,j,k+1}^n - u_{i+1,j,k-1}^n - u_{i-1,j,k+1}^n + u_{i-1,j,k-1}^n}{4h_x h_z} + \frac{v_{ij+1,k+1}^n - v_{ij+1,k-1}^n - v_{ij-1,k+1}^n + v_{ij-1,k-1}^n}{4h_z h_y}$$

$$i=1, \dots, N_x - 1, j=1, \dots, N_y - 1, k=1, \dots, N_z - 1$$

This scheme has a second-order error in all variables  $O(\tau^2) + O(h_x^2) + O(h_y^2) + O(h_z^2)$ . The stability of the scheme is ensured by the fulfillment of Courant's condition:

$$\frac{\tau^2 \mu}{h_x^2 + h_y^2 + h_z^2} \leq 1$$

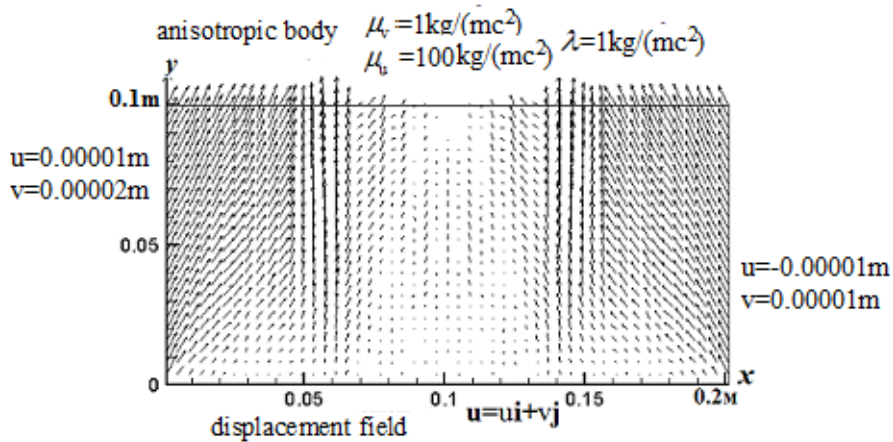


Figure 7- The field of displacements  $\mathbf{u} = u\mathbf{i} + v\mathbf{j}$  in an anisotropic body

Fig. 7 shows the displacement field according to Hooke's linear law  $m_u = 1, m_v = 1$  in an anisotropic body, in Fig. 8 in an isotropic body.

On the upper plane of the bar, the Neumann boundary condition  $\frac{\partial u}{\partial y} = 0, \frac{\partial v}{\partial y} = 0$ .

Obviously, the difference in the coefficients leads to different displacement fields, hence to different fields of internal stresses.

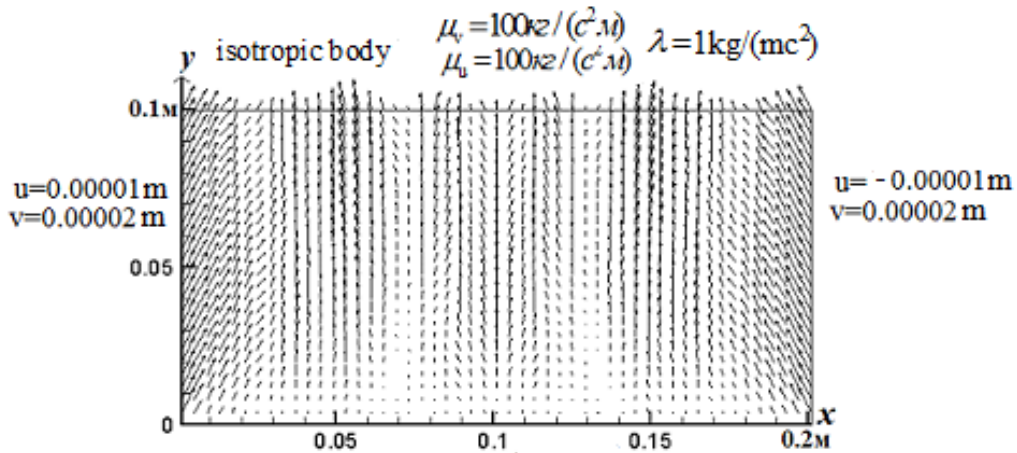


Figure 8 - The field of displacements in an isotropic body

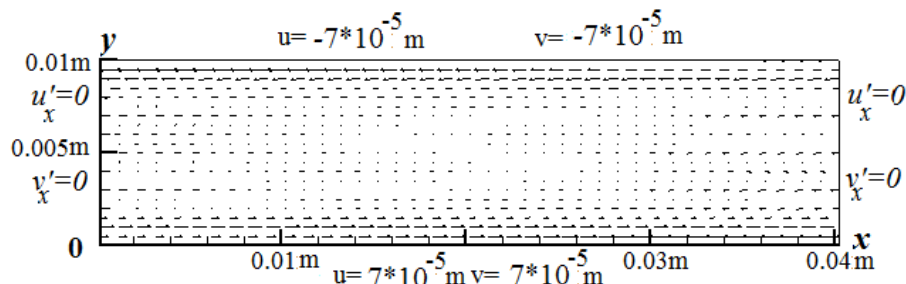


Figura 9 - Displacement field  $\mathbf{u} = u\mathbf{i} + v\mathbf{j}$   
by a nonlinear Hooke law of degree 3

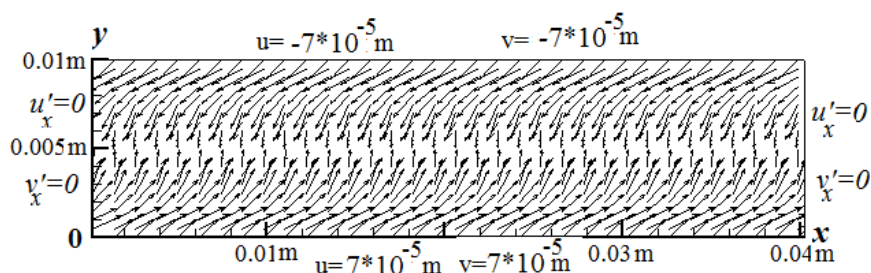


Figura 10 - Displacement field by a linear Hooke law

Fig. 9 shows the displacement field in an anisotropic body by Hooke's non-linear power law  $m_u = 3, m_v = 3$ . On the horizontal sides of the bar are given the Neumann boundary conditions are put on the lateral sides

$$\frac{\partial u}{\partial x} = 0, \frac{\partial v}{\partial x} = 0.$$

Lame coefficients in an anisotropic bar:

$$\lambda = 104.4 \text{ kg} / (\text{c}^2 \text{ m}), \mu_u = 80 \text{ kg} / (\text{c}^2 \text{ m}), \mu_v = 40 \text{ kg} / (\text{c}^2 \text{ m})$$

In Fig. 10 shows the displacement field in an anisotropic body according to Hooke's linear law  $m_u = 1, m_v = 1$ .

## CONCLUSIONS

The physical conclusions of the normal and tangential stresses prove the asymmetry of the stress tensor in a solid deformed body both for the isotropic Hooke's law and for an anisotropic, including nonlinear one. Concrete examples of numerical calculations of the state of an elastic body show the inadequacy and inconsistency of the hypothesis of the symmetry of the stress tensor of a continuous medium and, accordingly, of the equations of the Lamé elasticity theory.

The asymmetry of the stress tensor opens up wide possibilities for the modeling of displacements in a solid deformed body, which is shown by the application of Hooke's law in the anisotropic body of Fig. 7 and Hooke's nonlinear law of Fig. 9.

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**К. Б. Жақып-тегі**

ҚР БҒМ Математика және математикалық моделдеу институты, Алматы, Қазақстан  
Әл-Фараби атындағы ҚазҰУ

### **СЫЗЫҚСЫЗ ГУКТЫҢ ЗАҢЫ БІРТЕКТЕС ЕМЕС ЖӘНЕ АНИЗОТРОПТЫҚ ДЕНЕЛЕРДІҢ СЕРПІЛІМДІК ТЕОРИЯСЫНДА**

**Аннотация.** Тікелей сызықсыз Гук заңымен қатты майысқақ денелердің кернеулер тензорының компоненттері шығарылған. Ламенің екінші еселенуінің бағыттан тәуелділігі есептелген. Майысқақ қатты дененің серпілімдік теориясының кернеулер тензорының беттеспегендігі дәлелденген. Осыған сәйкес майысқақ қатты дененің сызықсыз серпілімдік теориясының теңдеулері жасалынған. Ламе гипотезасында толық емес жылжу дифференциалының беттескен жартысы қана пайдаланғаны көрсетілген, екінші антибеттескен жартысы лақтырылынған, соның салдарынан Ламе кернеулер тензорының беттескендігі шыққан. Жаңа теңдеулер үшін 2 ретті нақтылығы бар айқын схема жасалынған, соны пайдаланып жазық жолақтың серпілімдік күйі саналған, үстіңгі жақтауының ортасына жанама кернеулер және тік кернеулер әсер еткенде. Дәл сондай схема Ламе теңдеулеріне де қолдалынған. Саналған жылжулардың үлестірулік суреттері салыстырынып жатқан теңдеулердің айырмашылықтарын бейнелейді және Ламе теңдеулерінің майысқақ қатты дененің күйіне сәйкес еместігін көрсетеді. Ламе теңдеулерінің жалғандығы теориялық және физикалық тұрпатта бекітілген.

**Тірек сөздер:** анизотроптылық, созылу, кернеулер, тензор, теңдеулер.

УДК 539.2/.6

**К.Б. Джакупов**

Институт математики и математического моделирования МОН РК, Алматы, Казахстан  
Казахский Национальный Университет им.Аль-Фараби

### **НЕЛИНЕЙНЫЙ ЗАКОН ГУКА В ТЕОРИИ УПРУГОСТИ НЕОДНОРОДНЫХ И АНИЗОТРОПНЫХ ТЕЛ**

**Аннотация.** Непосредственно из физической связи с нелинейным законом Гука выводятся компоненты тензора напряжений твердого деформируемого тела и новые нелинейные уравнения теории упругости с несимметричным тензором напряжений, как частный случай получаются уравнения с линейным законом Гука. Гипотеза Ламе и уравнения Ламе не имеют физической связи с законом Гука, в этом заключается их фальшивость. Ламе взял за основу приближенную формулу неполного дифференциала и предположил в своей гипотезе пропорциональность компонент тензора напряжений симметричной половине данного неполного дифференциала смещения, причем антисимметричная половина дифференциала отбрасывается, следствием чего является фальшивая симметричность тензора напряжений Ламе. Новые нелинейные уравнения аппроксимируются явной схемой, с применением которой численно рассчитано упругое состояние плоского бруска при действующих на верхней грани нормальном и касательном напряжениях. Такая же схема применена для уравнений Ламе. Полученные картины распределения смещений наглядно демонстрируют различие решений сравниваемых систем уравнений упругости, а также несоответствие решения уравнений Ламе данному состоянию деформируемого тела. Теоретически и физически подтверждена фальшивость уравнений Ламе.

**Ключевые слова:** растяжение, касательное, нормальное, напряжения, тензор.



МАЗМУНЫ

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**ISSN 2518-1726 (Online), ISSN 1991-346X (Print)**

Редакторы *М. С. Ахметова, Т. А. Апендиев, Д. С. Аленов*  
Верстка на компьютере *А. М. Кульгинбаевой*

Подписано в печать 15.02.2018.  
Формат 60x88<sup>1</sup>/<sub>8</sub>. Бумага офсетная. Печать – ризограф.  
9 п.л. Тираж 300. Заказ 1.

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*Национальная академия наук РК*  
*050010, Алматы, ул. Шевченко, 28, т. 272-13-18, 272-13-19*