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ҚАЗАҚ ҰЛТТЫҚ УНИВЕРСИТЕТІНІҢ

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НАЦИОНАЛЬНОЙ АКАДЕМИИ НАУК  
РЕСПУБЛИКИ КАЗАХСТАН

КАЗАХСКИЙ НАЦИОНАЛЬНЫЙ  
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THE COMPLETENESS  
OF THE NONCOMMUTATIVE  $H_E(A, \ell_\infty)$  SPACE

**Abstract.** Considering the commutative case, we know that a maximal function  $f = \sup_n |f_n|$  belongs to  $L_p(\mu)$  if and only if there is a factorization  $f_n = cz_n = z_n c$  for all  $n \in \mathbb{N}$ , where  $c \in L_p(\mu)$  and  $\sup_n \|z_n\|_\infty < \infty$ . The theory of vector-valued noncommutative  $L_p$ -spaces are introduced first time by Pisier in 1998. Pisier considered the case  $M$  is hyperfinite. This theory solved maximal function's problem in noncommutative case. Later in 2002 Junge and Xu introduced general case. By using these noncommutative vector valued  $L_p$ -spaces Junge solved noncommutative version of Doob's maximal inequality problem in general case.

The noncommutative vector-valued Hardy spaces were introduced in [2]. In this paper, we consider maximal function's problems on noncommutative Hardy spaces. For this reason we introduce a noncommutative vector-valued symmetric Hardy space.

Our aim is discover their properties. It is presented another useful proof of completeness of this space. We also obtain factorization theorem like Saito's theorem. The work is mostly theoretical. The results can be used to further develop of noncommutative martingale theory, noncommutative ergodic theory, and operator valued Hardy spaces theory.

**Key words:** von Neumann algebra,  $\tau$ -measurable operator, subdiagonal algebra, noncommutative symmetric space, noncommutative Hardy space.

1 Preliminaries and Introduction

Let  $S(0,1)$  be the space of all measurable real-valued functions on  $(0,1)$  equipped with Lebesgue measure  $m$  (functions which coincide almost everywhere are considered identical).

For  $x \in S(0,1)$  we denote by  $\mu(x)$  the decreasing rearrangement of the function  $|x|$ . That is,

$$\mu(t, x) = \inf \{s \geq 0 : m(\{|x| > s\}) \leq t\}, \quad t > 0.$$

**Definition 1** We say that  $(E, \|\cdot\|_E)$  is a symmetric Banach function space if the following holds.

- (a)  $E$  is a subset of  $S(0,1)$ .
- (b)  $(E, \|\cdot\|_E)$  is a Banach space.
- (c) If  $x \in E$  and if  $y \in S(0,1)$  are such that  $|y| \leq |x|$ , then  $y \in E$  and  $\|y\|_E \leq \|x\|_E$ .
- (d) If  $x \in E$  and if  $y \in S(0,1)$  are such that  $\mu(y) = \mu(x)$ , then  $y \in E$  and  $\|y\|_E = \|x\|_E$ .

Furthermore we recall that the norm in  $E$  is said to be order continuous if, for every sequence  $\{x_n\}_{n \geq 0} \subset E$  such that  $x_n \downarrow 0$  in  $S(0,1)$ , we have that  $\|x_n\|_E \rightarrow 0$ . Order continuity of the norm is equivalent to separability of the space  $E$  (see [10]).

Special examples of such Banach function spaces are the spaces  $L_p(0,1)$ ,  $1 \leq p \leq \infty$ , equipped with their usual norm  $\|\cdot\|_p$ .

We recall that every symmetric Banach function space satisfies

$$L_\infty(0,1) \subset E \subset L_1(0,1)$$

with continuous embeddings.

We say that  $y$  is submajorized by  $x$  in the sense of Hardy-Littlewood (written  $y \ll x$ ) if

$$\int_0^t \mu(s, y) ds \leq \int_0^t \mu(s, x) ds, \quad t > 0.$$

Now let  $E$  be a Banach lattice. Let  $0 < r < \infty$ . Then  $E$  is said to be  $r$ -convex and  $r$ -concave, if there exists a constant  $C > 0$  such that for all finite sequence  $(x_n)$  in  $E$

$$\left\| \left( \sum_{k=1}^n |x_k|^r \right)^{1/r} \right\|_E \leq C \left( \sum_{k=1}^n \|x_k\|_E^r \right)^{1/r},$$

and

$$\left( \sum_{k=1}^n \|x_k\|_E^r \right)^{1/r} \leq C \left\| \left( \sum_{k=1}^n |x_k|^r \right)^{1/r} \right\|_E,$$

and as usual the best constant  $C > 0$  is denoted by  $M^{(r)}(E)$  and  $M_{(r)}(E)$ , respectively. We recall that for  $r_1 \leq r_2$  we have

$$M^{r_1}(E) \leq M^{r_2}(E)$$

and

$$M_{r_2}(E) \leq M_{r_1}(E).$$

For all needed information on convexity and concavity we once again refer to [10]. If  $M^{\max(1,r)}(E) = 1$ , then the  $r$ 'th power

$$E^r := \{x \in L_0(\Omega) : |x|^{1/r} \in E\}$$

endowed with the norm

$$\|x\|_{E^r} = \left\| |x|^{1/r} \right\|_E^r$$

is again a Banach function space which is  $1/\min(1, r)$ -convex.

Let  $\mathbf{H}$  be a Hilbert space. The closed densely defined linear operator  $x$  in  $\mathbf{H}$  with domain  $D(x)$  is said to be affiliated with  $\mathbf{M}$  if and only if  $uxu = x$  for all unitary operators  $u$  which belong to the commutant  $\mathbf{M}'$  of  $\mathbf{M}$ . An operator  $x$  affiliated with  $\mathbf{M}$  is said to be  $\tau$ -measurable, if for every  $\varepsilon > 0$  there exists a projection  $e$  in  $\mathbf{M}$  such that  $e(\mathbf{H}) \subseteq D(x)$  and  $\tau(1-e) < \varepsilon$ . The set of all  $\tau$ -measurable operators will be denoted by  $L_0(\mathbf{M})$ . The set  $L_0(\mathbf{M})$  is a  $*$ -algebra with sum and product being the respective closure of the algebraic sum product [12]. For each  $x$  on  $\mathbf{H}$  affiliated with  $\mathbf{M}$ , all spectral projection  $e_s^\perp(|x|) = \chi_{(s,\infty)}(|x|)$  corresponding to the interval  $(s;\infty)$  belong to  $\mathbf{M}$ , and  $x \in L_0(\mathbf{M})$  if and only if  $\chi_{(s,\infty)}(|x|) < \infty$  for some  $s \in \mathbf{R}$ . Recall the decreasing rearrangement (or generalized singular numbers) of an operator  $x \in L_0(\mathbf{M})$  is defined as follows

$$\mu(t, x) = \inf \{s > 0 : \lambda_s(x) \leq t\}, \quad t > 0$$

where

$$\lambda_s(x) = \tau(e_s^\perp(|x|)), s > 0.$$

The function  $s \mapsto \lambda_s(x)$  is called the distribution function of  $x$ . For more details on generalized singular value function of measurable operators we refer to [7]. Recall the construction of a Banach symmetric operator space  $L_E(M, \tau)$  (for convenience  $L_E(M)$ ). Let  $E$  be a Banach symmetric function space. Set

$$L_E(M, \tau) = \{x \in L_0(M, \tau) : \mu(x) \in E\}.$$

We equip  $L_E(M, \tau)$  with a natural norm

$$\|x\|_{L_E(M, \tau)} = \|\mu(x)\|_E, \quad x \in L_E(M, \tau).$$

It was further established in [15] that  $L_E(M, \tau)$  is Banach.

Since for each operator  $x \in L_0(M)$

$$\mu(|x|^r) = \mu(x)^r,$$

we conclude for every symmetric Banach function space  $E$  on the interval  $(0,1)$  which satisfies  $M^{\max(1,r)}(E) = 1$  that

$$L_{E^r}(M) := \{x \in L_0(M) : |x|^{1/r} \in L_E(M)\}$$

and

$$\|x\|_{L_{E^r}(M)} = \|\mu(|x|)\|_{E^r} = \|\mu(|x|^{1/r})\|_E^r = \left\| |x|^{1/r} \right\|_{L_E(M)}^r.$$

See [3, 5].

Let  $M$  be a finite von Neumann algebra on the Hilbert space  $H$  equipped with a normal faithful tracial state  $\tau$ . Let  $D$  be a von Neumann subalgebra of  $M$ , and let  $\Phi : M \rightarrow D$  be the unique normal faithful conditional expectation such that  $\tau \circ \Phi = \tau$ . A finite subdiagonal algebra of  $M$  with respect to  $\Phi$  is a  $w^*$ -closed subalgebra  $A$  of  $M$  satisfying the following conditions:

- (i)  $A + J(A)$  is  $w^*$ -dense in  $M$ ;
- (ii)  $\Phi$  is multiplicative on  $A$ , i.e.,  $\Phi(ab) = \Phi(a)\Phi(b)$  for all  $a, b \in A$ ;
- (iii)  $A \cap J(A) = D$ , where  $J(A)$  is the family of all adjoint elements of the element of  $A$ , i.e.,  $J(A) = \{a^* : a \in A\}$ .

The algebra  $D$  is called the diagonal of  $A$ . It is proved by Exel [6] that a finite subdiagonal algebra  $A$  is automatically maximal in the sense that if  $B$  is another subdiagonal algebra with respect to  $\Phi$  containing  $A$ , then  $B = A$ .

For brevity, we introduce the following definition which was defined in [1].

**Definition 2** Let  $E$  be a symmetric Banach space on  $(0,1)$  and  $A$  be a finite subdiagonal subalgebra of  $M$ . Then  $H_E(A) = [A]_{L_E(M)}$  called symmetric Hardy space associated with  $A$ , where  $[\cdot]_{L_E(M)}$  means closure in the norm of  $L_E(M)$ . We denote  $[A_0]_{L_E(M)}$  by  $H_E^0(A)$ .

The theory of vector-valued non-commutative  $L_p$ -spaces were introduced by Pisier in [11] for the case, when  $M$  is hyperfinite and Junge introduced these spaces for general setting in [8] (see also [4, 9]). The theory for the space  $L_E(M; \ell_\infty)$  was developed by Defant in [3] and Dirksen in [5] and in full analogy with the special case  $L_E = L_p$  considered in [4, 8, 9]. In fact, most of the basic results follow verbatim as soon as were replaced  $L_p$  by  $L_E$ ,  $L_{p'}$  by  $L_{E^x}$ , where  $1/p + 1/p' = 1$ , and  $L_{2p}$  by  $L_{E^{1/2}}$  in the proofs of those results.

Let us denote by  $L_E(\mathbf{M}; \ell_\infty)$  the space of all families  $x = (x_n)_{n \geq 1}$  in  $L_E(\mathbf{M}, \tau)$  for which there are operators  $a, b \in L_{E^{1/2}}(\mathbf{M})$  and a uniformly bounded sequence  $(y_n)_{n \geq 1}$  in  $\mathbf{M}$  such that there is a factorization  $x_n = ay_n b$  for all  $n \in \mathbf{N}$ . We set

$$\|x\|_{L_E(\mathbf{M}; \ell_\infty)} := \inf \left\{ \|a\|_{L_{E^{1/2}}(\square)} \sup_n \|y_n\|_\infty \|b\|_{L_{E^{1/2}}(\mathbf{M})} \right\},$$

where the infimum is taken over all such possible factorizations. Moreover, we denote by  $L_E(\mathbf{M}; \ell_\infty^{col})$  (here "col" should remind on the word "column") the space of all  $x = (x_n)_{n \geq 1}$  in  $L_E(\mathbf{M})$  for which there are  $b \in L_E(\mathbf{M})$  and a bounded sequence  $(y_n)_{n \geq 1}$  in  $\mathbf{M}$  such that  $x_n = y_n b$  for all  $n$ . We then put

$$\|x\|_{L_E(\mathbf{M}; \ell_\infty^{col})} := \inf \left\{ \sup_n \|y_n\|_\infty \|b\|_{L_E(\mathbf{M})} \right\}.$$

Similarly, the row version consisting of all families  $x = (x_n)_{n \geq 1}$  admitting a factorization  $x_n = ay_n$  with  $a \in L_E(\mathbf{M})$  and  $(y_n)_{n \geq 1}$  bounded in  $\mathbf{M}$  is denoted by  $L_E(\mathbf{M}; \ell_\infty^{row})$  and we define

$$\|x\|_{L_E(\mathbf{M}; \ell_\infty^{row})} := \inf \left\{ \|a\|_{L_E(\mathbf{M})} \sup_n \|y_n\|_\infty \right\}.$$

In both cases the infimum is again taken over all possible factorizations.

Now we define the analogue of this space by a similar way.

**Definition 3** We define  $H_E(\mathbf{A}; \ell_\infty)$  as the space of all sequences  $x = (x_n)_{n \geq 1}$  in  $H_E(\mathbf{A})$  which admit a factorization of the following form: there are  $a, b \in H_{E^{1/2}}(\mathbf{A})$ , and a bounded sequence  $y = (y_n) \subset \mathbf{A}$  such that

$$x_n = ay_n b, \forall n \geq 1. \tag{1}$$

Given  $x \in H_E(\mathbf{A}, \ell_\infty)$  define

$$\|x\|_{H_E(\mathbf{A}; \ell_\infty)} := \inf \left\{ \|a\|_{H_{E^{1/2}}(\mathbf{A})} \sup_n \|y_n\|_\infty \|b\|_{H_{E^{1/2}}(\mathbf{A})} \right\}, \tag{2}$$

where the infimum runs over all factorizations of  $(x_n)$  as above. Moreover, let us define  $H_E(\mathbf{A}; \ell_\infty^{col})$  as the space of all  $(x_n)_{n \geq 1}$  in  $H_E(\mathbf{A})$  for which there are  $b \in H_E(\mathbf{A})$  and bounded sequence  $(y_n)_{n \geq 1}$  in  $\mathbf{M}$  such that  $x_n = y_n b$  and

$$\|x\|_{H_E(\mathbf{A}; \ell_\infty)} := \inf \left\{ \sup_n \|y_n\|_\infty \|b\|_{H_E(\mathbf{A})} \right\}. \tag{3}$$

Similarly, we define the row version  $H_E(\mathbf{A}; \ell_\infty^{row})$  all sequences which allow a uniform factorization  $x_n = ay_n$ , again with  $a \in H_E(\mathbf{A})$  and  $(y_n)_{n \geq 1}$  uniformly bounded in  $\mathbf{M}$ .

This space with  $H_p(\mathbf{A}; \ell_\infty)$  was introduced in the paper [13, 14, 2] with some basic properties.

Section 1 contains some preliminary definitions. In section 2, we prove that  $H_E(\mathbf{A}, \ell_\infty)$  is Banach space.

## 2 Main results

**Theorem 1** Let  $E$  be an  $r$ -convex symmetric Banach space on  $(0,1)$  for some  $0 < r < \infty$ . Assume  $E$  does not contain  $c_0$ . Then  $H_E(\mathbf{A}, \ell_\infty)$  is Banach space.

*Proof.* Let us first check that  $\|\cdot\|_{H_E(\mathbf{A}, \ell_\infty)}$  satisfies triangle inequality. Let  $(h_n^{(1)}), (h_n^{(2)}) \in H_E(\mathbf{A}, \ell_\infty)$ , choose a factorization of  $h^{(j)}$  with  $j = 1, 2$ :

$$h_n^{(j)} = a^{(j)} x_n^{(j)} b^{(j)}, \forall n$$



such that

$$\|a^{(j)}\|_{H_{E^{1/2}}(A)} = \|b^{(j)}\|_{H_{E^{1/2}}(A)} = \|(h_n^{(j)})\|_{H_E(A, \ell_\infty)}^{\frac{1}{2}}$$

and

$$\sup_n \|x_n^{(j)}\|_\infty \leq 1 + \varepsilon, \quad j = 1, 2. \quad (4)$$

Indeed, for any  $\varepsilon > 0$  choose a factorization  $h_n^{(j)} = c^{(j)} y_n^{(j)} d^{(j)}$ ,  $\forall n \geq 1$  with  $j = 1, 2$  such that

$$c^{(j)} \in H_{E^{1/2}}(A), \quad d^{(j)} \in H_{E^{1/2}}(A), \quad \sup_n \|y_n^{(j)}\|_\infty = \alpha$$

and

$$\|(h_n^{(j)})\|_{H_E(A, \ell_\infty)} (1 + \varepsilon) \geq \left\| \alpha^{\frac{1}{2}} c^{(j)} \right\|_{H_{E^{1/2}}(A)} \sup_n \left\| \frac{y_n^{(j)}}{\alpha} \right\|_\infty \left\| \alpha^{\frac{1}{2}} d^{(j)} \right\|_{H_{E^{1/2}}(A)}. \quad (5)$$

Then by choosing

$$a^{(j)} = \frac{\alpha^{\frac{1}{2}} \|(h_n^{(j)})\|_{H_E(A, \ell_\infty)}^{1/2} c^{(j)}}{\left\| \alpha^{\frac{1}{2}} c^{(j)} \right\|_{H_{E^{1/2}}(A)}}, \quad b^{(j)} = \frac{\alpha^{\frac{1}{2}} \|(h_n^{(j)})\|_{H_E(A, \ell_\infty)}^{1/2} d^{(j)}}{\left\| \alpha^{\frac{1}{2}} d^{(j)} \right\|_{H_{E^{1/2}}(A)}}$$

and

$$x_n^{(j)} = \frac{\left\| \alpha^{\frac{1}{2}} c^{(j)} \right\|_{H_{E^{1/2}}(A)} \left\| \alpha^{\frac{1}{2}} d^{(j)} \right\|_{H_{E^{1/2}}(A)} y_n^{(j)}}{\alpha \|(h_n^{(j)})\|_{H_E(A, \ell_\infty)}},$$

we obtain

$$a^{(j)} x_n^{(j)} b^{(j)} = c^{(j)} y_n^{(j)} d^{(j)} = h_n^{(j)}, \quad j = 1, 2$$

and

$$\|a^{(j)}\|_{H_{E^{1/2}}(A)} = \|(h_n^{(j)})\|_{H_E(A, \ell_\infty)}^{1/2},$$

$$\|b^{(j)}\|_{H_{E^{1/2}}(A)} = \|(h_n^{(j)})\|_{H_E(A, \ell_\infty)}^{1/2}.$$

Let  $a^{(j)} = |a^{(j)*}| u^{(j)}$  and  $b^{(j)} = v^{(j)} |b^{(j)}|$  be the polar decompositions of  $(a^{(j)})^*$  and  $b^{(j)}$ , respectively. Then  $u^{(j)} x_n^{(j)} v^{(j)} \in M$ , so we can substitute  $x_n^{(j)}$  by  $u^{(j)} x_n^{(j)} v^{(j)}$  and therefore we are allowed to assume that the  $a^{(j)}$  and  $b^{(j)}$  are positive for  $j = 1, 2$ . Define operators:

$$a := (|a^{(1)*}|^2 + |a^{(2)*}|^2 + \varepsilon)^{\frac{1}{2}} \quad \text{and} \quad b := (|b^{(1)}|^2 + |b^{(2)}|^2 + \varepsilon)^{\frac{1}{2}};$$

clearly,

$$\begin{aligned} \|a\|_{E^2(A)}^2 &\leq \left( \|(a^{(1)*})\|_{E^2(A)}^2 + \|(a^{(2)*})\|_{E^2(A)}^2 + \varepsilon \right)^{\frac{1}{2}} \\ &= \left( \|(h_n^{(1)})\|_{H_E(A, \ell_\infty)} + \|(h_n^{(2)})\|_{H_E(A, \ell_\infty)} + \varepsilon \right)^{\frac{1}{2}}, \end{aligned}$$

a similar inequality holds for  $b$  with norm  $\|\cdot\|_E^{\frac{1}{2}}$ . By Remark 2.3 in [4] there exist contractions  $\omega^{(j)}, \theta^{(j)} \in M$  such that  $|(a^{(j)})^*| = a(\omega^{(j)})^*$ ,  $|b^{(j)}| = \theta^{(j)}b$  and

$$(\omega^{(1)})^* \omega^{(1)} + (\omega^{(2)})^* \omega^{(2)} = r(a^2), (\theta^{(1)})^* \theta^{(1)} + (\theta^{(2)})^* \theta^{(2)} = r(b^2).$$

And, since  $a^{-1}, b^{-1} \in M$  and  $(a^{-1})^{-1} = a, b \in L_E(M)$ , by Proposition 4.3. (i) in [1] there exist the unitary operators  $\nu^{(1)}, \nu^{(2)} \in M$  and  $w^{(1)}, w^{(2)} \in A$  such that  $a^{-1} = \nu^{(1)}w^{(1)}$  and  $b^{-1} = w^{(2)}\nu^{(2)}$ , where  $(w^{(1)})^{-1}, (w^{(2)})^{-1} \in H_{E^{1/4}}(A)$ . Obviously,

$$\begin{aligned} h_n^{(1)} + h_n^{(2)} &= (w^{(1)})^{-1}[(\nu^{(1)})^{-1}(\omega^{(1)})^* u^{(1)} x_n^{(1)} \nu^{(1)} \theta^{(1)} \\ &\quad + (\omega^{(2)})^* u^{(2)} x_n^{(2)} \nu^{(2)} \theta^{(2)} (\nu^{(2)})^{-1}](w^{(2)})^{-1}. \end{aligned}$$

Define the sequence

$$y_{(\cdot)} := (\nu^{(1)})^{-1}(\omega^{(1)})^* u^{(1)} x_{(\cdot)}^{(1)} \nu^{(1)} \theta^{(1)} + (\omega^{(2)})^* u^{(2)} x_{(\cdot)}^{(2)} \nu^{(2)} \theta^{(2)} (\nu^{(2)})^{-1}.$$

Since  $y_n = (w^{(1)})^{-1}[h_n^{(1)} + h_n^{(2)}](w^{(2)})^{-1} \in H_r(A)$  by Proposition 4.3. (ii) in [1]  $y_n \in H_r(A) \cap M = A$ . Consider for each fixed  $n$  the following mapping:

$$U : M_2(M) \rightarrow M_2(M)$$

defined by

$$U(X) = \begin{pmatrix} (\omega^{(1)})^* & (\omega^{(2)})^* \\ 0 & 0 \end{pmatrix} \begin{pmatrix} u^{(1)} & 0 \\ 0 & u^{(2)} \end{pmatrix} X \begin{pmatrix} \nu^{(1)} & 0 \\ 0 & \nu^{(2)} \end{pmatrix} \begin{pmatrix} \theta^{(1)} & 0 \\ \theta^{(2)} & 0 \end{pmatrix},$$

where

$$X = \begin{pmatrix} y_n^{(1)} & 0 \\ 0 & y_n^{(2)} \end{pmatrix} \in M_2(M).$$

We need to show that  $\|y_n\| \leq 1$ .

Indeed,

$$\begin{aligned} \|y_n\| &= \|(\nu^{(1)})^{-1}[(\omega^{(1)})^* u^{(1)} x_n^{(1)} \nu^{(1)} \theta^{(1)} + (\omega^{(2)})^* u^{(2)} x_n^{(2)} \nu^{(2)} \theta^{(2)}](\nu^{(2)})^{-1}\| \\ &= \|(\omega^{(1)})^* u^{(1)} x_n^{(1)} \nu^{(1)} \theta^{(1)} + (\omega^{(2)})^* u^{(2)} x_n^{(2)} \nu^{(2)} \theta^{(2)}\| = \|U(X)\| \\ &\leq \left\| \begin{pmatrix} (\omega^{(1)})^* & (\omega^{(2)})^* \\ 0 & 0 \end{pmatrix} \right\| \left\| \begin{pmatrix} u^{(1)} & 0 \\ 0 & u^{(2)} \end{pmatrix} \right\| \|X\| \left\| \begin{pmatrix} \nu^{(1)} & 0 \\ 0 & \nu^{(2)} \end{pmatrix} \right\| \left\| \begin{pmatrix} \theta^{(1)} & 0 \\ \theta^{(2)} & 0 \end{pmatrix} \right\| \\ &\leq \|\omega_1^* \omega_1 + \omega_2^* \omega_2\|^{\frac{1}{2}} \|\theta_1^* \theta_1 + \theta_2^* \theta_2\|^{\frac{1}{2}} \leq \|r(a^2)\|^{\frac{1}{2}} \|r(b^2)\|^{\frac{1}{2}} \leq 1. \end{aligned}$$

So,

$$\begin{aligned} \|(h_n^{(1)} + h_n^{(2)})\|_{H_E(A, \ell_\infty)} &\leq \|c\|_{H_{E^{1/2}}(A)} \sup_n \|y_n\| \|d\|_{H_{E^{1/2}}(A)} \\ &\leq (\|c^{(1)}\|^2\|_{H_{E^{1/4}}(A)} + \|c^{(2)}\|^2\|_{H_{E^{1/4}}(A)} + \varepsilon)^{\frac{1}{2}} (\|d^{(1)}\|^2\|_{H_{E^{1/4}}(A)} + \|d^{(2)}\|^2\|_{H_{E^{1/4}}(A)} + \varepsilon)^{\frac{1}{2}} \\ &\leq (\|c^{(1)}\|_{H_{E^{1/2}}(A)}^2 + \|c^{(2)}\|_{H_{E^{1/2}}(A)}^2 + \varepsilon)^{\frac{1}{2}} (\|d^{(1)}\|_{H_{E^{1/2}}(A)}^2 + \|d^{(2)}\|_{H_{E^{1/2}}(A)}^2 + \varepsilon)^{\frac{1}{2}} \\ &= \|(h_n^{(1)})\|_{H_E(A, \ell_\infty)} + \|(h_n^{(2)})\|_{H_E(A, \ell_\infty)} + \varepsilon. \end{aligned}$$

Then letting  $\varepsilon \rightarrow 0$ , we obtain the desired triangle inequality. To show the completeness, we take a Cauchy sequence  $(h^{(j)}) \in H_E(A, \ell_\infty)$  for which we may assume without loss of generality that for all  $k$

$$\| (h^{(j)} - h^{(j+1)}) \|_{H_E(A, \ell_\infty)} < \frac{2^{-3j}}{2}, \quad (j=1, 2, \dots).$$

Define for each  $N$  the sequences

$$\eta_{(c)}^N := \sum_{j=N}^{\infty} h_{(c)}^{(j+1)} - h_{(c)}^{(j)}$$

in  $H_E(A)$ .

First, we need to show that all of them belong to  $H_E(A, \ell_\infty)$  for all  $N$  and that

$$\| \eta^N \|_{H_E(A, \ell_\infty)} \leq 2^{-N+1}.$$

Let

$$h^{(j)} - h^{(j+1)} = a^{(j)} x_{(c)}^{(j)} b^{(j)}$$

with

$$\| a^{(j)} \|_{H_{E^{1/2}}(A)} \leq 2^{-j}, \| b^{(j)} \|_{H_{E^{1/2}}(A)} \leq 2^{-j}, \sup_n \| x_n^{(j)} \|_\infty \leq 2^{-j}.$$

As above, we may assume that  $a^{(j)}$  and  $b^{(j)}$  are positive, obviously

$$\sum_{j=1}^{\infty} | (a^{(j)})^* |^2 \in H_{E^{1/4}}(A), \quad \sum_{j=1}^{\infty} | b^{(j)} |^2 \in H_{E^{1/4}}(A)$$

and

$$\begin{aligned} \left\| \sum_{j=1}^{\infty} | (a^{(j)})^* |^2 \right\|_{H_{E^{1/4}}(A)} &\leq \sum_{j=1}^{\infty} \| | (a^{(j)})^* |^2 \|_{H_{E^{1/4}}(A)} = \\ &\sum_{j=1}^{\infty} \| | (a^{(j)})^* |^2 \|_{H_{E^{1/2}}(A)}^2 \sum_{j=1}^{\infty} \| a^{(j)} \|_{H_{E^{1/2}}(A)}^2 \leq \sum_{j=1}^{\infty} 2^{-2j} \leq 1; \end{aligned}$$

similarly,

$$\left\| \sum_{j=1}^{\infty} | b^{(j)} |^2 \right\|_{H_{E^{1/4}}(A)} \leq \sum_{j=1}^{\infty} \| | b^{(j)} |^2 \|_{H_{E^{1/2}}(A)}^2 = \sum_{j=1}^{\infty} \| b^{(j)} \|_{H_{E^{1/2}}(A)}^2 \leq 1.$$

Define  $a := (\sum_{j=1}^{\infty} | (a^{(j)})^* |^2 + \varepsilon)^{\frac{1}{2}} \in H_{E^{1/2}}(A)$  and  $b := (\sum_{j=1}^{\infty} | b^{(j)} |^2 + \varepsilon)^{\frac{1}{2}} \in H_{E^{1/2}}(A)$ , then

$$\begin{aligned} \| a \|_{H_{E^{1/2}}(A)} &= \left\| \sum_{j=1}^{\infty} | (a^{(j)})^* |^2 + \varepsilon \right\|_{H_{E^{1/4}}(A)}^{\frac{1}{2}} \\ &\leq \left( \left\| \sum_{j=1}^{\infty} | (a^{(j)})^* |^2 \right\|_{H_{E^{1/4}}(A)} + \varepsilon \right)^{\frac{1}{2}} = \sum_{j=1}^{\infty} \| a_j \|_{H_{E^{1/2}}(A)}^2 + \varepsilon \leq 1 + \varepsilon \end{aligned}$$

and

$$\| b \|_{H_{E^{1/2}}(A)} \leq 1 + \varepsilon.$$

So by letting  $\varepsilon \rightarrow 0$ , we obtain  $\| a \|_{H_{E^{1/2}}(A)} \leq 1$  and  $\| b \|_{H_{E^{1/2}}(A)} \leq 1$ . Let  $a^{(j)} = | (a^{(j)})^* | u^{(j)}$  and  $b^{(j)} = v^{(j)} | b^{(j)} |$ . On the other hand, according to Remark 2.3 in [4] there exist contractions  $\omega^{(j)}, \theta^{(j)} \in M$  such that  $| (a^{(j)})^* | = a (\omega^{(j)})^*$  and  $| b^{(j)} | = \theta^{(j)} b$  as above. Thus

$$\zeta_{(\cdot)}^N = \sum_{j=N}^{\infty} (\omega^{(j)})^* u^{(j)} x_{(\cdot)}^{(j)} v^{(j)} \theta^{(j)}.$$

Hence  $\sup_n \|\eta_n^N\|_{\infty} \leq 2^{-(N-1)}$ . So we obtain

$$\|\zeta_{(\cdot)}^N\| \leq \sum_{j=N}^{\infty} \|(\omega^{(j)})^* u^{(j)} x_{(\cdot)}^{(j)} v^{(j)} \theta^{(j)}\| \leq \sum_{j=N}^{\infty} \|x_{(\cdot)}^{(j)}\| \leq \sum_{j=N}^{\infty} 2^{-j} \leq 2^{-(N-1)}$$

and

$$\begin{aligned} \eta_n^N &= \sum_{j=N}^{\infty} a^{(j)} x^{(j)} b^{(j)} = \sum_{j=N}^{\infty} |(a^{(j)})^*| u^{(j)} x_n^{(j)} v^{(j)} |b^{(j)}| \\ &= \sum_{j=N}^{\infty} a(\omega^{(j)})^* u^{(j)} x_n^{(j)} v^{(j)} \theta^{(j)} b = a \left( \sum_{j=N}^{\infty} (\omega^{(j)})^* u^{(j)} x_n^{(j)} v^{(j)} \theta^{(j)} \right) b = a \zeta^N b. \end{aligned}$$

Hence

$$\eta^N \in H_E(A; \ell_{\infty})$$

and

$$\|\eta^N\|_{H_E(A; \ell_{\infty})} \leq \|a\|_{H_{E/2}(A)} \sup_n \|\zeta_n^N\|_{\infty} \|b\|_{H_{E/2}(A)} \leq 2^{-(N-1)}.$$

So

$$\left\| \sum_{k=1}^N (h^{(k+1)} - h^{(k)}) - \eta^1 \right\|_{H_E(A; \ell_{\infty})} = \|\eta^{N+1}\|_{H_E(A; \ell_{\infty})} \leq 2^{-N} \rightarrow 0,$$

as  $N \rightarrow \infty$

$$\sum_{k=1}^N (h^{(k+1)} - h^{(k)}) \rightarrow \eta^1.$$

Therefore we get  $h^{(N+1)} \rightarrow h^{(1)} + \eta^1$  conclusion.

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### КОММУТАТИВТІ ЕМЕС $H_E(A, \ell_\infty)$ КЕҢІСТІГІНІҢ ТОЛЫҚТЫҒЫ

**Аннотация.** Коммутативті жағдайды қарастыра отырып,  $f = \sup_n |f_n|$  максималды функциясы  $L_p(\mu)$  –ға тәуелді болса, сонда тек сонда ғана барлық  $n \in \mathbb{N}$  үшін  $f_n = cz_n = z_n c$  факторизациялауы бар болады, мұндағы  $c \in L_p(\mu)$  және  $\sup_n \|z_n\|_\infty < \infty$ . Вектор-мәнді коммутативті емес  $L_p$ -кеңістіктерінің теориясын 1998 жылы алғаш рет Писье енгізген. Писье  $M$ -гиперақырлы болған жағдайын қарастырған. Бұл теория коммутативті емес жағдайда максималды функция мәселесін шешті. Кейінірек 2002 жылы Юнге және Шүй жалпы жағдайда енгізген. Осы вектор-мәнді коммутативті емес  $L_p$ -кеңістіктерін пайдаланып, Юнге жалпы жағдайда Дубтың максималды теңсіздік мәселесінің коммутативті емес нұсқасын шешті.

Коммутативті емес вектор-мәнді Харди кеңістіктері [2]-де енгізілді. Осы мақалада Коммутативті емес Харди кеңістіктерінде максималды функцияның мәселелерін қарастырамыз. Сол себепті біз коммутативті емес вектор-мәнді симметриялық Харди кеңістіктерін енгіземіз.

Біздің мақсатымыз олардың қасиеттерін көрсету. Бұл кеңістіктің толықтығын дәлелдейтін тағы бір пайдалы дәлел келтірілген. Сондай-ақ, Сайто теоремасы ұқсайтын факторизациялық теореманы аламыз. коммутативті емес мартингал теориясы, коммутативті емес эргодик теориясы және оператор мәнді Харди кеңістіктер теориясы үшін қолдануға болады.

**Түйін сөздер:** Фон Нейман алгебрасы,  $\tau$  - өлшемді оператор, субдиагональді алгебра, коммутативті емес симметриялық кеңістік, коммутативті емес Харди кеңістігі.

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### ПОЛНОТА НЕКОММУТАТИВНОГО ПРОСТРАНСТВА $H_E(A, \ell_\infty)$

**Аннотация.** Рассматривая коммутативный случай, мы знаем, что максимальная функция  $f = \sup_n |f_n|$  принадлежит  $L_p(\mu)$  тогда и только тогда, когда существует факторизация  $f_n = cz_n = z_n c$  для всех  $n \in \mathbb{N}$ , где  $c \in L_p(\mu)$  и  $\sup_n \|z_n\|_\infty < \infty$ . Теория вектор-значных некоммутирующих  $L_p$ -пространств впервые вводится Письеом в 1998 году. Писье считал случай  $M$  гиперконечным. Эта теория решала задачу максимальной функции в некоммутирующем случае. Позднее в 2002 году Юнге и Сюй представили общий случай. Используя эти некоммутирующие векторнозначные  $L_p$ -пространства, Юнге решил некоммутирующий вариант максимальной задачи неравенства Дуба в общем случае.

Некоммутирующие векторнозначные пространства Харди были интродуцированы в [2]. В настоящей работе рассматриваются задачи максимальной функции на некоммутирующих пространствах Харди. По этой причине мы вводим некоммутирующее вектор-симметричное симметричное пространство Харди.

Наша цель - открыть их свойства. Представлено еще одно полезное доказательство полноты этого пространства. Мы также получаем теорему факторизации, такую как теорема Сайто. Работа в основном теоретическая. Результаты могут быть использованы для дальнейшего развития некоммутирующей теории мартингалов, некоммутирующей эргодической теории и операторнозначной теории пространств Харди.

**Ключевые слова:** алгебра Фон Неймана,  $\tau$ -измеримый оператор, поддиагональная алгебра, некоммутирующее симметричное пространство, некоммутирующее пространство Харди.

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