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THE STUDY OF THE NEUTRON HALO OF THE ¹¹BE NUCLEUS TAKING INTO ACCOUNT THE INFLUENCE OF AN EXTERNAL FIELD

Abstract. The aim of work is a theoretical study of the Coulomb breakup of halo nuclei in time-dependent quantum-mechanical approach. Exotic nuclei are the subject of intensive experimental research. Theoretical studies of Coulomb breakup of halo nuclei are relevant for the interpretation and planning of experiments for the study of light nuclei on radioactive beams. The investigations with beams of radioactive nuclei have opened new prospects in studying the structure of the atomic nucleus and have found wide applications in other areas of physics, including nuclear astrophysics. The halo is one of the most intensively studied objects in modern low-nucleus physics. A characteristic feature of halo nuclei physics is correlations between the mechanism of nuclear reaction and structure.

The breakup is one of the important tools for the theoretical study of the properties of halo nuclei. In these reactions, the information from the breakup of the projectile into fragments can be used to make a conclusion about the properties of the halo part of the wave function. With good approximation, the breakup could be considered as a transition from bound state of two (three) particles to the continuum, due to changing Coulomb field.

In this paper, the energy levels of the halo nucleus of ¹¹Be are calculated, taking into account the effect of an external magnetic field. The ¹¹Be nucleus is regarded as a neutron halo consisting of ¹⁰Be core and one neutron. This work is the initial stage of the work on the investigation of the breakup of halo nuclei in the quantum-mechanical approach.

Key words: Halo nucleus, Coulomb breakup, breakup cross section, exotic states of the nuclei, nonstationary Schrödinger equation, energy spectrum, nuclear potential.

Introduction. For the first time, nuclei with a neutron halo were discovered in 1985 by Tanikhata and et al. [1,2], where such exotic systems were tightly bound core and surrounded by a diffuse nuclear cloud. These systems were observed in the ground states (g.s.) of some light, neutron-rich radioactive nuclei located near the neutron stability boundary [3]. Previously it was believed that a halo could be formed only in radioactive nuclei located close to the neutron drip line. However, in the late 50's of last century, long before the discovery of the halo, A.I. Baz actually predicted [4] the possibility of its appearance even in stable nuclei near the neutron or proton emission thresholds. In particular, it was shown in [5] that the excited state of 3.09 (1/2⁻) MeV of the stable ¹³C nucleus could have a halo structure with an increased radius.

Coulomb breakup is one of the main tools for studying the halo nucleus. The cross section contains useful information about the structure of the halo. Thus, this topic is the subject of intense experimental and theoretical research. Among the halo nuclei, the ¹¹Be nucleus is of particular importance, since the relative simplicity of its structure allows more accurate theoretical studies. In fact, the bound states of the ¹¹Be nucleus can be described quite well as a ¹⁰Be nucleus and a weakly bound neutron. With a good approximation, the decay can be regarded as a transition from a two-particle bound state to a continuum due to a changing Coulomb field in the process of collision of nuclei with a target [6].

The neutron halo effect is caused by the presence of weakly bound states of neutrons located near the continuum. The small value of the binding energy of a neutron (or a group of neutrons) and the short-

range nature of nuclear forces lead to the tunneling of neutrons into the outer peripheral region over large distances from the core of the nucleus. In this case, the distribution density of peripheral neutrons is much smaller than the neutron distribution density inside the core [7].

Among the neutron halo nuclei, the nucleus ^{11}Be is of particular interest. In the simplest approximation, it can be considered as a two-particle system consisting of a ^{10}Be core and a weakly bound neutron. The halo-nucleus is described quite well by the wave function, which is the product of the wave functions of the core and the external halo. A number of experimental facts confirm that nucleons forming a nuclear halo have little effect on the core of the nucleus [6]. The most famous nuclei having the structure of a single-neutron halo are ^{11}Be , ^{11}Li , ^{17}C , ^{19}C and etc. [8]. They also have small binding energies, anomalously large sizes, narrow momentum distributions of fragments after the breakup, large interaction cross sections and electromagnetic dissociation.

The practical way of studying the halo structure is to investigate collisions of two nuclei with the transfer of energy and momentum. As a result, the transient properties of nuclear systems are studied in nuclear reactions, namely, the transition from the ground state to excited states [8]. The breakup is one of the important tools for the theoretical study of the properties of halo nuclei. In these reactions, the information from the breakup of the projectile into fragments can be used to make a conclusion about the properties of the halo part of the wave function. The Coulomb breakup is of particular interest, because the uncertainty about the assumption that the nuclear interaction between the projectile and the target plays an important role. Nevertheless, in order to correctly extract information from the cross sections, the accuracy of the description of the reaction mechanism must be established [9].

A characteristic feature of the physics of the halo nuclei is the close relationship between the mechanism of the nuclear reaction and the structure of the nucleus. The primary analysis [1,2] of experimental data on the cross sections of the interaction of nuclei with a halo has already led to the determination of the large material radii of these systems. Since in the known nuclei with a two-neutron halo the ground state is the only bound state, the breakup of nuclei with a halo in binary collisions is the final process of any reaction accompanied by excitation of the exotic system. The development of adequate models of the breakup is of great practical value as a means of extracting reliable information on the structure of nuclei with halo and the dynamics of the interaction processes [10].

In this paper, the influence of an external magnetic field on the ground state of the ^{11}Be nucleus is investigated, the splitting of the energy levels are calculated by numerical and analytical methods. The first order of perturbation theory was taken as an analytical method [11].

1. Numerical methods for solving the stationary Schrödinger equation. The problem is reduced to solving the stationary Schrödinger equation (SE):

$$H\psi_{Nlm} = E_N\psi_{Nlm} \quad (1)$$

The wave function could be written in the form:

$$\psi_{Nlm}(r) = R_{Nl}(r)Y_{lm}(\theta, \varphi) \quad (2)$$

where $Y_{lm}(\theta, \varphi)$ is a spherical function.

The Hamiltonian of the interaction [8]:

$$H_0(r) = -\frac{\hbar^2}{2\mu}\Delta + V_{cf}(r) \quad (3)$$

Then for the radial part $R_{Nl}(r)$ of the wave function we obtain the equation:

$$\left[-\frac{\hbar^2}{2\mu}\Delta + \frac{\hbar^2 l(l+1)}{2\mu r^2} + V_{cf}(r) \right] R_{Nl}(r) = E R_{Nl}(r) \quad (4)$$

where $\mu = \frac{m_n \cdot m_c}{M}$ reduced mass, m_n , m_c и $M = m_n + m_c$ – respectively, the masses of the neutron, core, and nucleus of ^{11}Be .

To solve the problem (4) it is more convenient to use the system of units, where E - energy, potential and mass are measured in the same energy units - MeV, and the radius of nucleus in fm, and $\hbar c = 197.328$ MeV · fm. Then equation (4) can be written in the form:

$$\left[-\frac{41,443}{2} \frac{d^2}{\left(\frac{\mu}{m_n}\right) dr^2} + \frac{41,443}{2} \frac{l(l+1)}{\left(\frac{\mu}{m_n}\right) r^2} + V_{cf}(r) \right] R_l(r) = E R_l(r) \quad (5)$$

The potential $V_{cf}(r)$ consists of a central term and a spin-orbit interaction term taking into account the spin of the neutron \mathbf{I} and the angular momentum \mathbf{L} of the relative motion of the neutron-core [6,9].

$$V_{cf}(r) = V_0(r) + \mathbf{LIV}_{LI}(r) \quad (6)$$

$V_{cf}(r)$ is an internal interaction between the nucleus and the fragment of the projectile [9]. The central potential of equation (6):

$$V_0(r) = -V_l f(r, R_0, a) \quad (7)$$

where the Woods-Saxon form factor:

$$f(r, R_0, a) = \left[1 + \exp\left(\frac{r-R_0}{a}\right) \right]^{-1} \quad (8)$$

Spin-orbit interaction is expressed as [9]:

$$V_{LI}(r) = V_{LS} \frac{1}{r} \frac{d}{dr} f(r, R_0, a) \quad (9)$$

The values of the potential parameters are given in Table 1, they are chosen as in [6].

Table 1 - Potential parameters

$V_{l=0}$ (MeV)	$V_{l>0}$ (MeV)	V_{LS} (MeV fm ²)	a (fm)	R_0 (fm)
59.5	40.5	32.8	0.6	2.669

Here V_l is the depth of the Woods-Saxon potential, a is the diffuseness, and R_0 is the radius of the ^{11}Be ($R_0 = 1.2A^{1/3}$ fm). The standard value V_{LS} is used for the potential depth l_s for the p-shell core [1].

We seek the solution of the SE (5) under boundary conditions using numerical methods of the inverse iteration [12], finite-difference [13] and the sweep methods [12].

$$\begin{cases} R_{Nl}(r) \rightarrow const, r \rightarrow 0 \\ R_{Nl}(r) \rightarrow 0, r \rightarrow \infty \end{cases} \quad (5)$$

The method of reverse iteration is characterized by sufficiently rapid convergence to the solution. The accuracy of the result must be checked on the residual. The accuracy of the computational scheme is equal to $\Delta_i = |E^{(i)} - E^{(i-1)}| < 10^{-6}$. In the equation, the second-order derivative can be simplified for a computer scheme using finite-difference approximation, described in detail in paper [7]. A radial grid is introduced over r_j on the interval $r \in [0, r_m]$, for convenience we have introduced the notation $R(r_j) = R_j$. The wave function for the first iteration will be found by sweep method, then the normalization will be checked. Thus, the energy level will be found. Negative energy states are normalized and describe either the physical bound states of the nucleus ^{11}Be or states forbidden by the Pauli principle [9].

1.1 Numerical methods for solving the stationary Schrödinger equation.

1.1.1 The results: energy spectrum of d^1Be . The stationary Schrödinger equation (radial part) is solved by the method of reverse iteration [12]. The solution scheme is as follows:

$$\begin{cases} \widehat{A}\vec{R} = E\vec{R} \\ (\widehat{A} - \widehat{I}E^{(0)})\vec{R}^{(i)} = \vec{R}^{(i-1)}, i = \overline{1, l_{max}} \\ E^{(i)} = E^{(0)} + \frac{1}{\widehat{R}^{(i)}, \widehat{R}^{(i-1)}} \end{cases} \quad (11)$$

where $E^{(0)}$ is the initial approximation for the energy, i - is the iteration number, $\widehat{R}^{(0)}$ - is the initial vector, and the calculated finite vector $\widehat{R}^{(i)}$ - is normalized at each iteration $\widehat{R}(r) = \widehat{\varphi}^{(i_{max})}$.

The advantage of this method is that the final answer will not depend on the choice of the initial approximation, since the answer quickly converges. Nevertheless, the accuracy of the result must be checked on the residual.

From equation (11) that the accuracy of the computational scheme is

$$\Delta_i = |E^{(i)} - E^{(i-1)}| < 10^{-6} \quad (12)$$

or the discrepancy is $\delta_i < 10^{-6}$:

$$(\widehat{A} - \widehat{I}E^{(i)})R^{(i)} = \delta_i \quad (13)$$

1.1.2 The sweep method. We seek the solution SE (5) in the form (11) under the boundary conditions:

$$\begin{cases} R_{Nl}(r) \rightarrow const, r \rightarrow 0 \\ R_{Nl}(r) \rightarrow 0, r \rightarrow \infty \end{cases} \quad (14)$$

In the equation there is a second-order differential, which can be simplified for a computational circuit using the finite-difference method, described in detail in [13]:

$$\frac{d^2}{dr^2} (R_j^{(1)}) = \frac{R_{j+1}^{(1)} - 2R_j^{(1)} + R_{j-1}^{(1)}}{h^2} \quad (15)$$

Here we introduce a radial mesh with respect to r_j , where h is the step along the grid r_j , for convenience, we have introduced the notation $R(r_j) = R_j$.

The Schrodinger equation goes to the following form

$$\hat{c}_j \vec{R}_{j+1}^{(1)} + \hat{d}_j \vec{R}_j^{(1)} + \hat{e}_j \vec{R}_{j-1}^{(1)} = \vec{R}_j^{(0)} \quad (16)$$

It can be seen that equation (16) consists of a three-diagonal matrix. The solution will be solved in the following form, using the sweep method [12]:

$$\begin{aligned} \bar{\psi}_j &= \alpha_j \bar{\psi}_{j+1} + \beta_j \\ \bar{\psi}_{j-1} &= \alpha_{j-1} \bar{\psi}_j + \beta_{j-1} \end{aligned} \quad (17)$$

Substituting $\bar{R}_{j-1} = \alpha_{j-1} \bar{R}_j + \beta_{j-1}$ into equation (16), we find that

$$\bar{R}_j = \alpha'_j \bar{R}_{j+1} + \beta'_j$$

where the coefficients are:

$$\alpha'_j = -(\hat{d}_j + \alpha_{j-1} \hat{e}_j)^{-1} \cdot \hat{c}_j$$

$$\beta_j' = (\hat{d}_j + \alpha_{j-1} \hat{e}_j)^{-1} (\vec{R}_j^{(0)} - \beta_{j-1} \cdot \hat{e}_j) \quad (18)$$

Using this scheme, at first the coefficients α_j' and β_j' are found (direct sweep), then the radial wave function $\vec{R}_j^{(1)}$ is found by reverse sweep. Further, the normalization is checked. Thus, there is a wave function for the first iteration. Further, as described above, the energy level is found. Negative energy states are normalized and describe either the physical bound states of the projectile or states forbidden by the Pauli principle [9].

1.2 Results:the energy spectrum of ^{11}Be . Applying these numerical methods, in this paper, the energy levels of the ^{11}Be nucleus for the Woods-Saxon potential were reproduced as a test program as in [6,9]. The ^{11}Be nucleus is regarded as a neutron halo consisting of a ^{10}Be core and one neutron [6,9]. As a result, energy levels were obtained for the ground and first excited states. These data are given in Table 2 and compared with the results of [9].

Table 2 - The energies of the ground and excited states of ^{11}Be

J^π	l	$E_{\text{exp.}}(\text{MeV})$ [14]	$E_{\text{theor.}}(\text{MeV})$ [9]	$E_{\text{theor.}}(\text{MeV})$ (this work)
$\frac{1}{2}^+$	0	-0.503	-0.5013	-0.5013
$\frac{1}{2}^-$	1	-0.183	-0.1844	-0.1844

As already stated above, for a finite-difference approximation of a second-order equation with respect to a radial variable r , a grid was used on the interval $r \in [0, r_m]$, where $r_m = 800$ fm for the ground and first excited states [9]. The convergence of the computational scheme for $\Delta r \rightarrow 0$ is presented in Table 3, where N_r is the number of points, Δr is the step along the radial grid, and E is the energy of the bound state.

Table 3- Convergence of a computational scheme on a homogeneous radial mesh

N_r	Δr	$E, l=0$	N_r	Δr	$E, l=1$
2000	0.4	-0.501318	2000	0.4	-0.184423
4000	0.2	-0.780709	4000	0.2	-0.1883722
8000	0.1	-0.845679	8000	0.1	-0.1903396
16000	0.05	-0.861629	16000	0.05	-0.1913216

2 The splitting of the energy levels of ^{11}Be due to the influence of an external magnetic field (Zeeman splitting). In this chapter, the effect of an external magnetic field on the halo state of the ^{11}Be nucleus is studied, i.e. the splitting of the energy levels by a numerical method is calculated and numerical results are compared with the analytical solution. The first order of perturbation theory was chosen as an analytical method [11].

Under the influence of an external magnetic field, the magnetic moments of the nuclei are oriented in a certain way and it becomes possible to observe transitions between the nuclear energy levels associated with these different orientations: transitions occurring under the action of radiation of a certain frequency. The quantization of the energy levels of the nucleus is a direct consequence of the quantum nature of the angular momentum of nucleus, assuming $2I + 1$ values. The spin quantum number (spin) I can take any value that is a multiple of $\frac{1}{2}$. The splitting of energy levels in a magnetic field can be called a nuclear Zeeman splitting, since it is analogous to the splitting of electron levels in a magnetic field (the Zeeman effect) [11].

Nuclear magnetic resonance (NMR) based on the Zeeman effect is widely used in nuclear spectroscopy. At present, it is difficult to indicate a field in the natural sciences, where NMR has not been used to some extent. NMR spectroscopy methods are widely used in chemistry, molecular physics, biology, agronomy, medicine and in the study of natural formations, etc. Devices for the investigation of the entire human body by methods of magnetic resonance (by NMR tomography methods) have been developed and are being manufactured [16].

Let's write the radial Schrödinger equation (5) adding an external field ΔV_μ :

$$\left[\frac{\hbar^2}{2m} \frac{d^2}{dr^2} + \frac{\hbar^2 l(l+1)}{2mr^2} + V(r) + \Delta V_\mu \right] R_l(r) = ER_l(r) \quad (19)$$

We rewrite the equation, corrected for our system of units:

$$\left[\frac{\hbar^2}{2mr_0^2 E_0} \frac{d^2}{dr^2} + \frac{\hbar^2 l(l+1)}{2\mu r_0^2 E_0} - \frac{V_0'}{1 + \exp\left(\frac{r-R}{a}\right)} + \frac{\Delta V_\mu}{E_0 r_0} \right] R_l\left(\frac{r}{r_0}\right) = \frac{E}{E_0} R_l\left(\frac{r}{r_0}\right) \quad (20)$$

$$\left[-\frac{k_1}{2} \frac{d^2}{dr^2} + \frac{k_1 l(l+1)}{2r^2} + V(r) + \Delta V_\mu \right] R_l(r) = E_l R_l(r), \quad (21)$$

where the additional potential ΔV_μ describes the interaction of the neutron spin with an external magnetic field, since, as mentioned above, the neutron halo nucleus ^{11}Be is considered as system of $^{10}\text{Be} + n$ and is defined as $\Delta V = \mathbf{B} \cdot \boldsymbol{\mu}_n \cdot \hat{S}_n$; correction factors for the nuclear system of units $k_1 = 41,443$ and $k_2 = 3.15 \cdot 10^{-13} \frac{\text{MeV}}{\text{Gfm}}$, here B - is the strength of the magnetic field, μ_n - is the magnetic moment of the neutron, \hat{S}_n - is the projection of the spin on the axis. Since the neutron spin is $s=1/2$, the projection of the spin on the selected direction takes two values: $+1/2$ and $-1/2$. In the (21), the wave function $R_l(r)$ must be replaced by the spin wave function $R_l(r) \rightarrow R_l(r) \cdot \chi_m$, where χ_m are two-component spinors, and the spin operators are 2×2 matrices. For the case when the field is directed along the z axis: $\hat{S}_z = \pm \frac{1}{2} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$.

In the representation $|s m_s\rangle$, where the projection of the spin to the axis takes the values $m_s = +\frac{1}{2}, m_s = -\frac{1}{2}$ then the basis vectors of this representation have the form [14]

$$\chi_{\frac{1}{2}, \frac{1}{2}} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \chi_{\frac{1}{2}, -\frac{1}{2}} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

Introducing the spin wave function, it can be seen that the SE is splitted into two equations

$$\begin{aligned} E'_{lm=\frac{1}{2}} &= E_l + \Delta E_{m=\frac{1}{2}} \\ E'_{lm=-\frac{1}{2}} &= E_l + \Delta E_{m=-\frac{1}{2}} \end{aligned} \quad (22)$$

so the level shifts are defined as:

$$\begin{aligned} \Delta E_{m=\frac{1}{2}} &= \langle R_{lm}^{(r)} | \frac{1}{2} k_2 \cdot \mathbf{B} \cdot \boldsymbol{\mu}_n | R_{lm}^{(r)} \rangle \\ \Delta E_{m=-\frac{1}{2}} &= \langle R_{lm}^{(r)} | -\frac{1}{2} k_2 \cdot \mathbf{B} \cdot \boldsymbol{\mu}_n | R_{lm}^{(r)} \rangle \end{aligned} \quad (23)$$

The same can easily be calculated to the case when the field is directed along the x or y axes.

Next, the program was modified to calculate the energy shifts by introducing the spinor, so the number of matrices and vectors are doubled.

Stationary Schrödinger Equation

$$H_0 R_l(r) = E R_l(r) \quad (24)$$

can be rewritten in the following form:

$$\sum_{j=1}^M H_{ij}^{(0)} R_j = \sum_j \delta_{ij} E R_j = E R_i \quad (25)$$

In order to describe the nuclear interaction, we used the Woods-Saxon potential with the parameters given in the first chapter and the potential of the Gauss shape to verify the technique [14]:

$$V(r) = V_0 e^{-\left(\frac{r}{r_0}\right)^2} = V_0 e^{-gr^2} \quad (26)$$

For $l = 0$, the potential depth is chosen as for the Woods-Saxon potential $V_0 = -59.5$ MeV, the potential width $g = \frac{1}{r_0^2} = 0.117 \text{ fm}^{-2}$.

Figure 1 shows the potentials of Woods-Saxon (WS) and Gauss (G) of the ground state as a function of the radial coordinate. According to the graph, it can be used the interval from 0 to $r_m = 8 \text{ fm}$.

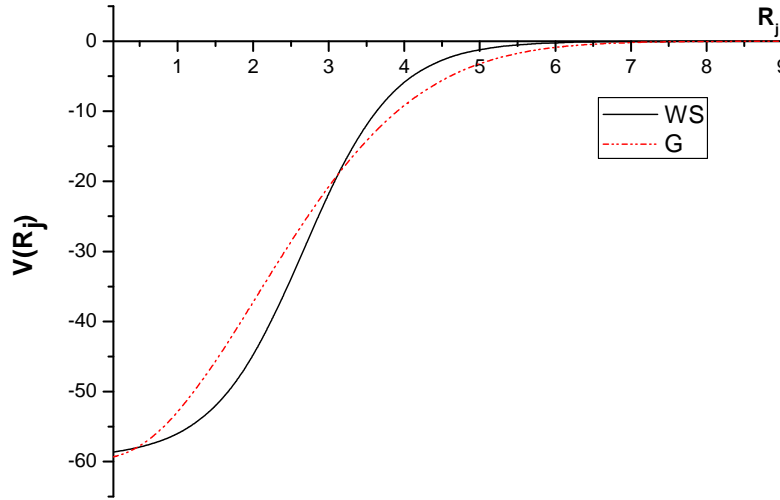


Figure 1 - The shape of the Woods-Saxon and Gauss potential for the ground state of ^{11}Be as a function of the radial variable

The results for the ground state are shown in Table 4. The numerical results are compared with the analytical solution, and the first order of perturbation theory is chosen as the analytical one [12]. The SE, taking into account the perturbation, is written as:

$$(H_0 + \Delta V) R_0(r) = E'_0 R_0(r) \quad (27)$$

$$E'_0 = E_0 + \Delta E$$

The energy shifts in perturbation theory are calculated as:

$$\Delta E_{\frac{1}{2}} = \int_0^\infty R_0(r) \Delta V_{\frac{1}{2}}(r) R_0(r) dr$$

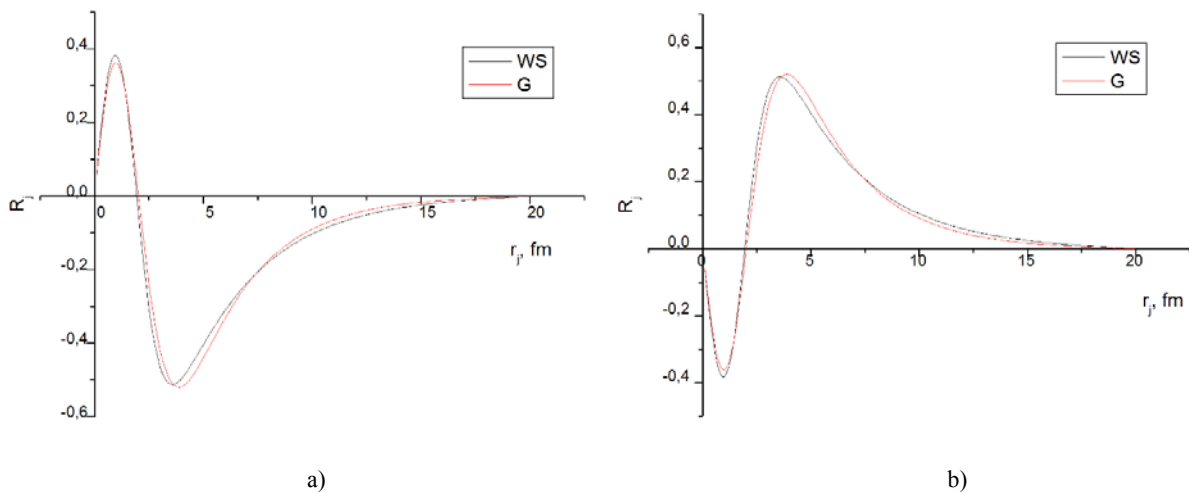
$$\Delta E_{-\frac{1}{2}} = \int_0^\infty R_0(r) \Delta V_{-\frac{1}{2}}(r) R_0(r) dr \quad (28)$$

The field strength varied from 0.1 to 2000 Gauss; it can be seen that the results are in good agreement with the analytical ones.

Table 4 - The energy shift of the ground state of ^{11}Be due to the influence of an external magnetic field

$R_m=8$ $M=200$	$\Delta E_{\text{pert}}(B_z)$ perturbation	$\Delta E_{\text{num}}(B_z)$ Gauss num.	$\Delta E_{\text{num}}(B_z)$ WS num.	$\Delta E_{\text{pert}}(B_z)$ perturbation	$\Delta E_{\text{num}}(B_z)$ Gaussnum.	$\Delta E_{\text{num}}(B_z)$ WSnum.
B (Gauss)	$m_s=+1/2$ spin projection			$m_s= - 1/2$ spin projection		
0.1	0.0003	0.0003	0.0003	-0.0003	-0.0003	-0.0003
1	0.0030	0.0030	0.0030	-0.0030	-0.0030	-0.0030
10	0.0300	0.0301	0.0301	-0.0300	-0.0300	-0.0300
100	0.3008	0.3008	0.3008	-0.3008	-0.3008	-0.3008
200	0.6016	0.6016	0.6016	-0.6016	-0.6016	-0.6016
300	0.9024	0.9025	0.9025	-0.9024	-0.9025	-0.9025
400	1.2033	1.2033	1.2033	-1.2033	-1.2033	-1.2033
500	1.5041	1.5041	1.5041	-1.5041	-1.5041	-1.5041
1000	3.0082	3.0082	3.0082	-3.0082	-3.0082	-3.0082
2000	6.0165	6.0165	6.0165	-6.0165	-6.0165	-6.0165

Figure 2 shows the wave functions of the s-state of ^{11}Be for the spin projection of $+1/2$ (Fig. a) and $-1/2$ (Fig. b). Black is denoted for the Woods-Saxon (WS) potential, redline is Gauss (G). When the magnetic field is varied, the wave functions do not change.

Figure 2 - a) Radial wave function when the spin is directed upwards ($+1/2$) and b) when the spin is directed downward ($-1/2$)

Conclusion. Using numerical methods, in this paper, the energy levels of the ^{11}Be nucleus were reproduced as a test program using the Woods-Saxon potential for describing the nuclear interaction as in [6,9]. The ^{11}Be nucleus is regarded as a neutron halo consisting of a ^{10}Be core and one neutron [6,9].

Also, the energy level shifts were calculated due to the influence of the magnetic field, using two different potentials: the Woods-Saxon and Gauss forms. The numerical results coincide with the analytical solution, and the first order of perturbation theory is chosen as the analytical one.

This work is the initial stage of work on the investigation of the breakup of halo nuclei in the quantum-mechanical approach. A detailed investigation is planned to research the effect of the external field on the breakup of the halo nucleus, using the numerical method for solving the nonstationary SE.

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ИЗУЧЕНИЕ НЕЙТРОННОГО ГАЛО ЯДРА ^{11}Be С УЧЕТОМ ВЛИЯНИЯ ВНЕШНЕГО ПОЛЯ

Аннотация. Целью работы является теоретическое исследование процессов кулоновского развала гало ядер в рамках нестационарного квантово-механического подхода. Экзотические ядра являются предметом интенсивного экспериментального исследования. Теоретические исследования кулоновского развала гало ядер актуальны для интерпретации и планирования экспериментов по изучению легких ядер на радиоактивных пучках. Исследования с пучками радиоактивных ядер открыли новые перспективы в изучении структуры атомного ядра и нашли широкие приложения в других областях физики, включая ядерную астрофизику. Гало ядра являются одним из наиболее интенсивно исследуемых объектов в современной малонуклонной ядерной физике. Характерной особенностью физики ядер с гало является тесная взаимосвязь механизма ядерной реакции и структуры.

Развал является одним из важных инструментов для изучения свойств гало ядер. В этих реакциях, информация, поступающая от диссоциации снаряда на фрагменты может быть использована, чтобы сделать вывод о свойствах гало части волновой функции. С хорошим приближением, развал гало ядра можно рассматривать как переход от связанного состояния двух (трех) частиц к континууму, в связи с изменяющимся кулоновским полем.

В данной работе рассчитаны энергетические уровни гало ядра ^{11}Be , с учетом влияния внешнего магнитного поля, т.е. вычислено расщепление энергетических уровней численным и аналитическим методами с использованием двух разных потенциалов: в форме Вудс-Саксона и Гаусса. Ядро ^{11}Be , имеющее как нейтронное гало, состоящий из кора ^{10}Be и одного нейтрона. Эта работа является начальным этапом работы по исследованию развала гало ядер в кванто-механическом подходе.

Ключевые слова: гало ядро, кулоновский развал, сечение развала, экзотические состояния ядер, стационарное уравнение Шредингера, энергетический спектр, ядерный потенциал.

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