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CRITERIA FOR STRONG CONVERGENCE OF SOLUTIONS SINGULARLY OF THE PERTURBED CAUCHY PROBLEM

Abstract: The first-order integral equation with a non-smooth right-hand side is solved by the deviating argument method. It is shown that the solution of the corresponding singularly perturbed Cauchy problem converges to the solution of a non-perturbed problem. The criteria of strong convergence and the spectral properties of the auxiliary problem are used. The square root of a class of Sturm-Liouville operators is found.

Keywords: strong convergence, spectrum, spectral decomposition, square root of the operator, equation with deviating argument, Hilbert-Schmidt theorem.

1. Introduction

We consider a singularly perturbed Cauchy problem

$$\begin{aligned} &L_\varepsilon y = \varepsilon y'(\mathrm{x}) + a\, \mathrm{y}(\mathrm{x}) = \mathrm{f}(\mathrm{x}), \mathrm{x} \in (0,1] \\ &y(0) = 0 \end{aligned} \qquad (1.1)\text{-}(1.2)$$

in space $L^2(0,1)$, where a is the complex constant, $\varepsilon > 0$ is a small parameter, and $f(\mathrm{x}) \in L^2(0,1)$.

For what values of the constant a the solution of problem (1.1) - (1.2) converges strongly to the solution of the problem $L_0 y_0 = a y_0(x) = f(x)$, for $\varepsilon \to +0$?

The solution of problem (1.1) - (1.2) has the form $y(x,\varepsilon,f) = L_\varepsilon^{-1} f(x)$, hence, we are talking about the strong convergence of the sequence of operators L_ε^{-1} to the operator $L_0^{-1} = I/a$, where is the I unit operator. Since the right-hand side of (1.1) is not assumed to be smooth, we mean the strong solutions of problem (1.1) - (1.2), and (1.1) is understood almost everywhere in (0.1).

To clarify the formulation of the problem, we give the corresponding definitions [1, c.11].

Definition 1.1. Let's $\{A_n\}$ a sequence of bounded operators. It is said that this sequence converges uniformly to the operator A , if $\|A_n - A\| \to 0$ for $n \to \infty$.

Definition 1.2. A sequence $\{A_n\}$ of linear operators (generally speaking, unbounded) with a common domain of definition is called strongly convergent on D (to the operator) if for any $u \in D$

$$\|A_n u - Au\| \to 0 \text{ at } n \to \infty.$$

Definition 1.3 A sequence $\{A_n\}$ is called weakly convergent (to A) if for any $u \in D$ sequence the $\{A_n u\}$ sequence converges weakly to Au , in other words, this means that $(A_n u, \psi) \to (Au, \psi)$ for any $u \in D$ and any $\psi \in H = L^2(0,1)$.

The inverse operator L_ε^{-1} of problem (1.1) - (1.2) has the form $L_\varepsilon^{-1} f(x) = \frac{1}{\varepsilon}\int_0^x f(x) e^{-\frac{a}{\varepsilon}(x-t)} dt$ and, obviously, this operator is completely continuous in the space $H = L^2(0,1)$. If $\left\| L_\varepsilon^{-1} - L_0^{-1} \right\| \to 0$ for $n \to \infty$, then the operator L_0^{-1} is also completely continuous. Since it is not so, uniform convergence is not possible. If $a > 0$, then, weak convergence holds, which was proved in [2]. The main difference between the formulation of the problem and the known singularly perturbed problems [3-12] is that we do not require the right-hand side of a certain smoothness, so all known methods are not applicable in this situation. Our method is based on the spectral decomposition of Volterra operators [13-17].

The aim of the paper is to establish a criterion for strong convergence of a sequence of operators L_ε^{-1} to an operator L_0^{-1}, for $\varepsilon \to +0$.

2. Research methods

In our investigations we rely on the following Banach-Steinhaus theorem.

The Banach-Steinhaus theorem. In order that the sequence of linear bounded operators A_n strongly converges, it is necessary and sufficient that the norms of the operators A_n be uniformly bounded, and that the sequences $A_n x$ be convergent for all x of some dense in H the set [18, c.129].

Consider in space $H = L^2(0,1)$ the Sturm-Liouville boundary value problem

$$\begin{aligned} -y''(x) &= \mu y(x); \\ y(0) &= 0, y'(1) + by(1) = 0, \end{aligned} \qquad (2.1)\text{-}(2.2)$$

where $b - const, b > 0; \mu -$ is the spectral parameter.

If $\mu = 0,$, then the general solution of equation (2.1) has the form $y(x) = A + Bx,$ where $A, B -$ const. Substituting this expression into the boundary conditions (2.2), we obtain $A = 0; B + b \cdot B = 0, B(1 + \text{b}) = 0,$ since $b > 0, B = 0$, I e $y(\text{x}) \equiv 0$.

If $\mu \neq 0$, then the general solution of equation (2.2) has the form,

$$y(x,\mu) = A\cos\sqrt{\mu}x + \frac{B\sin\sqrt{\mu}x}{\sqrt{\mu}}(\mu \neq 0), \qquad (2.3)$$

Where $A, B -$ arbitrary constants depend (in general) on the spectral μ parameter. Substituting (2.3) into the boundary conditions (2.2), we obtain a system of equations with respect to unknown constants A, B.

Consequently, the eigenvalues of the Sturm-Liouville operator are the squares of the roots of the characteristic function

$$\Delta(\mu) = \cos\sqrt{\mu} + b \cdot \frac{\sin\sqrt{\mu}}{\sqrt{\mu}} = 0 \ . \qquad (2.4)$$

For real values b the problem (2.1) - (2.2) is self-adjoint, therefore all the zeros of the function (2.4) are real, that is, if μ_0 it is a zero of a function $\Delta(\mu)$, then it is a real quantity, then $\sqrt{\mu_0}$ a real or purely imaginary quantity.

If $\cos\sqrt{\mu_0}=0$, then $\sin\sqrt{\mu_0}=\pm 1$, so $\Delta(\lambda_0)=\pm\frac{b}{\sqrt{\mu_0}}\neq 0$; if $\sin\sqrt{\mu_0}=0$, then $\Delta(\mu_0)=\pm 1\neq 0$;,

if $\mu_0=0$, then $\Delta(0)=1+b>1$. Using these circumstances, we transform equations to a form convenient for investigation.

$$\Delta(\mu)=\frac{b\cos\sqrt{\mu}}{\sqrt{\mu}}\cdot\left(tg\sqrt{\mu}+\frac{\sqrt{\mu}}{b}\right)=0 .$$

Thus, the zeros of the function $\Delta(\lambda)$ coincide, with nonzero zeros of the function $F\left(\sqrt{\mu}\right)=tg\sqrt{\mu}+\sqrt{\mu}/b$.

Assuming, $\nu=\sqrt{\mu}$ for convenience, we investigate the zeros of functions $F(\nu)=tg\nu+\nu/b$ which, as we have already noted, are real or purely imaginary. Since the function $F(\nu)$ is odd, if ν_0 it is a zero of this function, then it $-\nu_0$ is also its zero, and since $(-\nu_0)^2=\nu_0^2$, the "negative" zeros of the functions $F(\nu)$ do not give new eigenvalues, so it suffices to restrict the right half-axis $\nu>0$ and the upper part of the imaginary axis $\nu=i\tau(\tau>0)$.

If $\nu=i\tau(\tau>0)$, then $F(i\tau)=tgi\tau+\frac{i\tau}{b}=\left(th\tau+\frac{\tau}{b}\right)i\neq 0$ the function $\Delta(\mu)$ does not have negative zeros, in other words, the Sturm-Liouville problem (2.1) - (2.2) does not have negative eigenvalues.

If $\nu>0$, then $F(\mathrm{v})=tg\nu+\frac{\nu}{b}$, and it is obvious that this function does not vanish in those intervals, where $tg\nu\geq 0$, i.e. when $n\pi\leq\nu\leq n\pi+\frac{\pi}{2}, n=0,1,2,\ldots$, therefore, we study the intervals $n\pi-\frac{\pi}{2}\leq\nu\leq n\pi, n=1,2,\ldots$.

It is obvious that $F(\mathrm{n}\,\pi-\frac{\pi}{2})=-\infty, \mathrm{F}(\mathrm{n}\,\pi)=\frac{n\pi}{b}>0$, when $n=1,2,\ldots$, the derivative of the function in this interval $F(\mathrm{v})$ is positive, since $F'(\mathrm{v})=\frac{1}{\cos^2\nu}+\frac{1}{b}>\frac{1}{b}>0$,. Consequently, in the interval $\left(n\pi-\frac{\pi}{2},n\pi\right)n=1,2,\ldots$ exactly one root of the equation is contained $F(\mathrm{v})=0$. Thus, if $b>0$, then the function $F(\mathrm{v})$ has no imaginary roots, i.e. all its roots are real, while the positive roots are localized in intervals

$$n\pi-\frac{\pi}{2}<\nu_n<n\pi, n=1,2,\ldots; F(\nu_n)=0$$

Now we investigate the behavior of these roots $\nu_n=\nu_n(\mathrm{b}),\mathrm{n}=1,2,\ldots$ for $b\to+\infty$. From the equation $F(\nu_n)=0$, we have.

$$tg\nu_n=-\nu_n/b,(\mathrm{n}=1,2,\ldots),\to\mathrm{v}_n=n\pi+arctg\left(-\frac{\nu_n}{b}\right)=n\pi-arctg\frac{\nu_n}{b}$$

Consequently, $\lim\limits_{b\to+\infty}\nu_n(\mathrm{b})=\mathrm{n}\,\pi-\mathrm{arctg}\,0=\mathrm{n}\,\pi,\mathrm{n}=1,2,\ldots$.

When the parameter b is changed, the roots $v_n(b)(n=1,2,...)$ do not stick together, as can be seen from inequality

$$v_{n+1}(b)-v_n(b)=(n+1)\pi-arctg\frac{v_{n+1}(b)}{b}-n\pi+arctg\frac{v_n}{b}=\pi+arctg\frac{v_n(b)}{b}-arctg\frac{v_{n+1}(b)}{b}>\pi+0-\frac{\pi}{2}\geq\frac{\pi}{2}.$$

Consequently, $\inf\limits_{b>0,m,n}\left|v_n(b)-v_m(b)\right|\geq\frac{\pi}{2}, m,n=1,2,...$.

We estimate the rates of aspiration to its boundary values $v_n(b), n=1,2,...$ of the roots at $b\to+\infty$. By Lagrange's formula [19, p.226], we have

$$F(n\pi)-F(v_n)=\frac{n\pi}{b}-0=F'(\xi_n)(n\pi-v_n)=\left(\frac{1}{b}+\frac{1}{\cos^2\xi_n}\right)(n\pi-v_n),$$

$$n\pi-v_n=\frac{\frac{n\pi}{b}}{\frac{1}{b}+\frac{1}{\cos^2\xi_n}}=\frac{n\pi}{1+\frac{b}{\cos^2\xi_n}}, v_n(b)<\xi_n<n\pi,$$

$$0<n\pi-v_n<\frac{n\pi}{1+b}, \forall b>0.$$

Now we find the normalized eigenfunctions of the boundary-value problem (2.1) - (2.2) corresponding to the eigenvalues $\mu_n=v_n^2, \left(n\pi-\frac{\pi}{2}\right)^2<\mu_n<(n\pi)^2, n=1,2,...$. It is not difficult to see that the eigenfunctions have the form $\varphi_n(x)=B_n\cdot\sin v_n x, n=1,2,...,$ where B_n the normalized coefficients, we calculate these coefficients.

$$\|\varphi_n(x)\|^2=B_n^2\cdot\int_0^1\sin^2 v_n x dx=B_n^2\cdot\int_0^1\frac{1-\cos 2v_n x}{2}dx=\frac{B_n^2}{2}\left[x-\frac{\sin 2v_n x}{2v_n}\right]\Big|_0^1=\frac{B_n^2}{2}\left[1-\frac{\sin 2v_n}{2v_n}\right]=1;$$

Consequently
$$B_n(b)=\sqrt{\frac{2}{1-\frac{\sin 2v_n}{2v_n}}}, \tag{2.5}$$

So $\lim\limits_{b\to+\infty}B_n(b)=\sqrt{2}$

Let's find the limit $\lim\limits_{b\to+\infty}\varphi_n(x)$. By the Lagrange formula

$$\sin n\pi x-\sin v_n x=x\cdot\cos\xi_n(n\pi-v_n), v_n<\xi_n<n\pi; n=1,2,...$$

$$\left|\sin n\pi x-\sin v_n x\right|\leq|x|(n\pi-v_n)<\frac{|x|n\pi}{1+b}, \forall b>0, \Rightarrow\left\|\sin n\pi x-\sin v_n x\right\|<\frac{n\pi}{1+b}\|x\|\leq\frac{n\pi}{1+b}, \forall b>0. \tag{2.6}$$

Then, by (2.5) - (2.6), we obtain that $\left\|\varphi_n(x)-\sqrt{2}\,\mathrm{sinn}\,\pi x\right\|\to 0$, for $b\to+\infty$.

Indeed ,

$$\left\|\varphi_n(x)-\sqrt{2}\sin n\pi x\right\|=\left\|B_n\sin v_n x-\sqrt{2}\sin n\pi x\right\|=\left\|B_n\sin v_n x-\sqrt{2}\sin v_n x+\sqrt{2}\sin v_n x-\sqrt{2}\,\mathrm{sinn}\,\pi x\right\|\leq$$

$$\leq\left|B_n-\sqrt{2}\right|\left\|\sin v_n x\right\|+\sqrt{2}\left\|\sin v_n x-\mathrm{sinn}\,\pi x\right\|\leq\left|B_n-\sqrt{2}\right|+\frac{\sqrt{2}n\pi}{1+b}\to 0 \text{ at } b\to+\infty$$

Since the boundary value problem (2.1) - (2.2) is self-adjoint, its normalized eigenfunctions $\{\varphi_n(x)\}, n=1,2,...$ form an orthonormal basis of the space $L^2(0,1)$. We have proved the following lemma.

Lemma 2.1. If $b>0$, then the Sturm-Liouville boundary value problem

$$-y''(x)=\mu y(x), x\in(0,1);$$
$$y(0)=0, y'(1)+b\cdot y(1)=0 \qquad (2.1)\text{-}(2.2)$$

has an infinite set of positive eigenvalues $\mu_n\,(n=1,2,...)$ localized in the intervals $\left(n\pi-\frac{\pi}{2}\right)^2<\mu_n<(n\pi)^2, n=1,2,...$

Own functions: $\varphi_n(x)=B_n\cdot\sin\sqrt{\mu_n}x, B_n(b)=\sqrt{2/\left(1-\frac{\sin 2v_n}{2v_n}\right)}, n=1,2,...,$

corresponding to these eigenvalues, form an orthonormal basis of the space $H=L^2(0,1)$.

The following equalities hold:

1) $\lim\limits_{b\to+\infty}\sqrt{\mu_n(b)}=n\pi;$ (2.7)

2) $\lim\limits_{b\to+\infty}B_n(b)=\sqrt{2};$ (2.8)

3) $\left\|\varphi_n(x)-\sqrt{2}\sin n\pi x\right\|\to 0$ при $b\to+\infty$, где $n=1,2,...$. (2.9)

Lemma 2.2. (about the square root). If b is an arbitrary complex constant, and λ is an eigenvalue of the Sturm-Liouville boundary value problem

$$Lz=-z''+b^2z=\lambda z(x), x\in(0,1); \qquad (2.10)$$

$$z(0)=0, z'(1)+bz(1)=0, \qquad (2.11)$$

then the quantity $\pm\sqrt{\lambda}$ is an eigenvalue of the Cauchy problem

$$\begin{cases} Bz=z'(x)+bz(x)=\pm\sqrt{\lambda}z(1-x), x\in(0,1] \\ z(0)=0 \end{cases} \qquad (2.12)\text{-}(2.13)$$

and conversely, if $\pm\sqrt{\lambda}$ it is an eigenvalue of the Cauchy problem (2.12) - (2.13), then $z(x)$ it is an eigenfunction of the boundary-value problem (2.10) - (2.11) corresponding to the eigenvalue λ.

Evidence.

Assuming $\mu=\lambda-b^2$ from (2.10), we have $-z''(x)=\mu z(x)$. The general solution of this equation has the form

$$z\left(x,\sqrt{\mu}\right)=A\cos\sqrt{\mu}x+\frac{B\sin\sqrt{\mu}x}{\sqrt{\mu}}, A,B-const. \qquad (2.14)$$

If $\mu=0$, then $z(x,0)=A+Bx$, then from the boundary conditions (2.11) we obtain

$$z(x,0)\big|_{x=0}=A=0,\Rightarrow z(x,0)=Bx,\Rightarrow z'(x,0)=B,$$
$$\left[z'(x,0)+b\cdot z(x,0)\right]_{x=1}=B+b\cdot B=B(1+b)=0,\Rightarrow B=0, z(x,0)\equiv 0.$$

If $\mu\neq 0$, then substituting (2.14) into the boundary conditions (2.11), we obtain

$$z\left(x,\sqrt{\mu}\right)\Big|_{x=0}=A=0,\Rightarrow z\left(x,\sqrt{\mu}\right)=B\cdot\frac{\sin\sqrt{\mu}x}{\sqrt{\mu}}(\mu\neq 0),\Rightarrow z'\left(x,\sqrt{\mu}\right)=B\cdot\cos\sqrt{\mu}x,\Rightarrow B\cos\sqrt{\mu}+b\cdot\frac{B\sin\sqrt{\mu}}{\sqrt{\mu}}=$$

$$B\cdot\left(\cos\sqrt{\mu}+b\cdot\frac{\sin\sqrt{\mu}}{\sqrt{\mu}}\right)=0$$

Thus, the eigenfunctions of the Sturm-Liouville problem (2.10) - (2.11) have the form

$$z\left(x,\sqrt{\mu}\right)=\frac{B\cdot\sin\sqrt{\mu}x}{\sqrt{\mu}}, \tag{2.15}$$

Where $\left\{\sqrt{\mu}\right\}$ - are the roots of equation

$$\cos\sqrt{\mu}+b\cdot\frac{\sin\sqrt{\mu}}{\sqrt{\mu}}=0. \tag{2.16}$$

By virtue of formulas (2.15) - (2.16), we have

$$z\left(1-x,\sqrt{\mu}\right)=\frac{B\sin\sqrt{\mu}(1-x)}{\sqrt{\mu}}=B\cdot\frac{\sin\sqrt{\mu}\cos\sqrt{\mu}x-\cos\sqrt{\mu}\sin\sqrt{\mu}x}{\sqrt{\mu}}=B\cdot\frac{\sin\sqrt{\mu}}{\sqrt{\mu}}\cos\sqrt{\mu}x-\cos\sqrt{\mu}\cdot\frac{B\sin\sqrt{\mu}}{\sqrt{\mu}}=$$

$$=-B\frac{\cos\sqrt{\mu}}{b}\cos\sqrt{\mu}x-\cos\sqrt{\mu}-z\left(x,\sqrt{\mu}\right)=-\cos\sqrt{\mu}\cdot\left[\frac{B\cos\sqrt{\mu}}{b}+z\left(x,\sqrt{\mu}\right)\right]=-\cos\sqrt{\mu}\cdot\left[\frac{B\cos\sqrt{\mu}}{b}+z\left(x,\sqrt{\mu}\right)\right]=-\cos\sqrt{\mu}\left[\frac{z'\left(x,\sqrt{\mu}\right)}{b}+z\left(x,\sqrt{\mu}\right)\right],$$

$$b\cdot z\left(1-x,\sqrt{\mu}\right)=-\cos\sqrt{\mu}\cdot[z'\left(x,\sqrt{\mu}\right)+b\cdot z\left(x,\sqrt{\mu}\right),\Rightarrow z'\left(x,\sqrt{\mu}\right)+b\cdot z\left(x,\sqrt{\mu}\right)=-\frac{b}{\cos\sqrt{\mu}}\cdot z\left(1-x,\sqrt{\mu}\right); \tag{2.17}$$

From equation (2.16), we have

$$tg\sqrt{\mu}=-\sqrt{\mu}/b,\Rightarrow 1+tg^2\sqrt{\mu}=1+\frac{\mu}{b^2}=\frac{b^2+\mu}{b^2}=\frac{\lambda}{b^2},\frac{1}{\cos^2\sqrt{\mu}}=\frac{\lambda}{b^2},$$

$$\cos^2\sqrt{\mu}=\frac{b^2}{\lambda},\lambda=\frac{b^2}{\cos^2\sqrt{\mu}},\pm\sqrt{\lambda}=\frac{b}{\cos\sqrt{\mu}},\Rightarrow z'\left(x,\sqrt{\mu}\right)+bz\left(x,\sqrt{\mu}\right)=\sqrt{\lambda}z\left(1-x,\sqrt{\mu}\right).$$

It is obvious from the boundary condition (2.11) that $z(0)=0$.

Note that $\lambda=\sqrt{\mu+b^2}$, there fore $\lambda=\pm\sqrt{\mu+b^2}$, and the signs before the radical are chosen according to the rule $\cos\sqrt{\mu}=\pm\frac{b}{\sqrt{\mu+b^2}}$

Conversely, if (2.12) - (2.13) holds, then putting $x=1$ in (2.12), we obtain $z'(1)+b-z(1)=0$. Differentiating equations (2.12), we obtain

$$z''(x)+bz'(x)=\mp\sqrt{\lambda}z'(1-x)=\mp\sqrt{\lambda}\left[\pm\sqrt{\lambda}z(x)-bz(1-x)\right]=-\lambda z(x)\pm\sqrt{\lambda}bz(1-x),\Rightarrow$$

$$z''(x)+b\cdot\left[\pm\sqrt{\lambda}z(1-x)-bz(x)\right]=-\lambda z(x)\pm bz(1-x),z''(x)-b^2z(x)=-\lambda z(x)$$

$$-z''(x)+b^2z(x)=\lambda z(x).$$

Corollary 2.1. If $\varphi_n(x)$ the normalized eigenfunction of the Sturm-Liouville problem (2.10) - (2.11), then we have the formula

$$\varphi_n'(x)\cdot\cos\sqrt{\mu_n}=-b\cdot\left[\varphi_n(1-x)+\varphi_n(x)\cdot\cos\sqrt{\mu_n}\right], \tag{2.18}$$

where are μ_n eigenvalues, and $\varphi_n(x)$ the eigenfunctions of the Sturm-Liouville problem:

$$-\varphi_n''(x)=\mu_n\varphi_n(x);\ z_n(0)=0\ ,\ z_n'(1)+bz_n(1)=0\ .$$

The formula (2.18) is a simple consequence of formula (2.17).

Lemma 2.3.If b the real quantity is different from zero, then we have the formula

$$e^{-bx}=\sum_{n=1}^{\infty}(e^{-bx},\varphi_n)*\varphi_n(x)=-\sum_{n=1}^{\infty}\frac{\varphi_n(1)}{b}\cos\sqrt{\mu_n}*\ln(x), \tag{2.19}$$

where $\varphi_n(x)$ (n = 1,2, ...) are orthonormal eigenfunctions of the boundary value problem:

$$-y''(x)=\mu y(x),x\in(0,1);$$

$$y(0)=0,y'(1)+by(1)=0$$

Evidence.

$$(e^{bx},\varphi_n)=\int_0^1 e^{bx}\varphi_n(x)dx=\int_0^1\frac{\varphi_n(x)de^{bx}}{b}=\frac{\varphi_n(x)e^{bx}}{b}\Big|_0^1-\int_0^1\frac{e^{bx}\varphi_n'(x)}{\cos\sqrt{\mu_n}}dx=\left\|-\frac{\varphi_n'(x)}{b}=\frac{\varphi_n(1-x)}{\cos\sqrt{\mu_n}}+\varphi_n(x)\right\|=$$

$$=\frac{\varphi_n(1)e^b}{b}+\int_0^1\frac{e^{bx}\varphi_n(1-x)}{\cos\sqrt{\mu_n}}dx+\int_0^1 e^{bx}\varphi_n(x)dx=\frac{\varphi_n(1)e^b}{b}+\int_0^1\frac{e^{b(1-x)}\varphi_n(x)}{\cos\sqrt{\mu_n}}dx+(e^{bx},\varphi_n),$$

$$\frac{\varphi_n(1)e^b}{b}+\int_0^1\frac{e^{b(1-x)}\varphi_n(x)}{\cos\sqrt{\mu_n}}dx=0,\Rightarrow\frac{\varphi_n(1)}{b}=-\int_0^1\frac{e^{-bx}\varphi_n(x)}{\cos\sqrt{\mu_n}}dx=-\frac{(e^{-bx},\varphi_n)}{\cos\sqrt{\mu_n}}.$$

Consequently

$$e^{-bx}=\sum_{n=1}^{\infty}(e^{-bx},\varphi_n)*\varphi_n(x)=-\sum_{n=1}^{\infty}\frac{\varphi_n(1)\cos\sqrt{\mu_n}}{b}*\varphi_n(x).$$

Lemma 2.4. If a and $\varepsilon(\varepsilon\neq 0)$ are arbitrary complex constants, then the operators

$$A_\varepsilon y=\varepsilon y'(x)+ay(x);D(A_\varepsilon)=\{y(x)\in C'(0,1)\cap C[0,1],y(0)=0\},$$

and

$$B_\varepsilon z=\varepsilon z'(x)+\mathrm{Re}\,az(x);D(B_\varepsilon)=\{z(x)\in C'(0,1)\cap C[0,1],z(0)=0\}$$

similar to each other and the similarity operator is the multiplication operator

$$V_\varepsilon f(x)=e^{-i\frac{jma}{\varepsilon}x}f(x)$$

Evidence.

Assuming, $y=V_\varepsilon z=e^{-i\frac{jma}{\varepsilon}x}\cdot z(x)$, we have $y(0)=z(0)=0$, and

$$A_\varepsilon y=\varepsilon\cdot[e^{-i\frac{jma}{\varepsilon}x}z'(x)-\frac{jma}{\varepsilon}i\cdot e^{-i\frac{jma}{\varepsilon}x}z(x)]+a*e^{-i\frac{jma}{\varepsilon}x}z(x)=e^{-i\frac{jma}{\varepsilon}x}[\varepsilon-z'(x)-$$

$$-jma\cdot i\cdot z(x)+\mathrm{Re}\,a\cdot z(x)+ijma\cdot z(x)]=e^{-i\frac{jma}{\varepsilon}x}[\varepsilon z'(x)+\mathrm{Re}\,a\cdot z(x)]=V_\varepsilon B_\varepsilon z(x)\Rightarrow$$

$$A_\varepsilon V_\varepsilon z=V_\varepsilon B_\varepsilon z,\Rightarrow V_\varepsilon^{-1}A_\varepsilon V_\varepsilon=B_\varepsilon$$

Corollary 2.2. The norms of operators A_ε^{-1} B_ε^{-1} are the same. In fact,

$B_\varepsilon^{-1}=V_\varepsilon^{-1}A_\varepsilon^{-1}V_\varepsilon,\Rightarrow\|B_\varepsilon^{-1}\|\le\|V_\varepsilon^{-1}\|\cdot\|A_\varepsilon^{-1}\|\cdot\|V_\varepsilon^{-1}\|\le\|A_\varepsilon^{-1}\|$, similarly from equality,

$A_\varepsilon^{-1}=V_\varepsilon B_\varepsilon^{-1}V_\varepsilon^{-1}$, имеем $\|A_\varepsilon^{-1}\|\le\|B_\varepsilon^{-1}\|$ we have, therefore, $\|A_\varepsilon^{-1}\|=\|B_\varepsilon^{-1}\|$.

3. Research results

Theorem 3.1. If $y(x,\varepsilon,f)$ is a solution of a singularly perturbed Cauchy problem

$$L_\varepsilon y = \varepsilon y'(x) + ay(x) = f(x), x \in [0,1] \tag{3.1}$$

$y(0) = 0$, (3.2) where a -const,> 0, f (x), $a>0$,f(x) $\in L^2(0,1)$, then the expansion

$$y(x,\varepsilon,f) = -\frac{1}{a}\sum_{n=1}^{\infty} (Sf,\phi_n)\cos\sqrt{\mu_n} * \varphi_n(x), \tag{3.3}$$

where $Sf(x) = f(1-x), \mu_n$ are the eigenvalues of the Sturm-Liouville problem

$$-y''(x) = \mu - y(x), x \in (0.1); \tag{3.4}$$

$$y(0) = 0, y'(1) + \frac{a}{\varepsilon} y(1) = 0, \tag{3.5}$$

A $\varphi_n(x)$-normalized eigenfunctions of this problem corresponding to these eigenvalues.

Evidence.

Let there be $y_N(x,\varepsilon,f)$ a partial sum of the series (3.3), then, by virtue of formula (2.18), we have

$$y_N(x,\varepsilon,f) = -\frac{1}{a}\sum_{n=1}^{N} (st,\phi_n)\cos\sqrt{\mu_n} * \phi_n(x),$$

$$y'_N(x,\varepsilon,f) = -\frac{1}{a} * \sum_{n=1}^{N} (st,\phi_n) * \cos\sqrt{\mu_n} * \phi'(x) =$$

$$= (\cos\sqrt{\mu_n} * \phi'_n(x) = -\frac{a}{\varepsilon}.[\phi_n(1-x) + \phi_n * \cos\sqrt{\mu_n}] = -\frac{1}{a}\sum_{n=1}^{N} (sf,\phi_n)$$

$$\{-\frac{a}{\varepsilon}[\phi_n(1-x) + \phi_n(x)\cos\sqrt{\mu_n}]\} =$$

$$= \frac{1}{\varepsilon} * \sum_{n=1}^{N} (st,\phi_n) * [\phi_n(1-x) + \phi_n(x)\cos\sqrt{\mu_n}],$$

$$\varepsilon y_N(x,\varepsilon,f) = \sum_{n=1}^{N} (sf,\phi_n) * \phi_n(1-x) + \sum_{n=1}^{N} (st,\phi_n)\cos\sqrt{\mu_n}\phi_n(x) =$$

$$= \sum_{n=1}^{N} (f,S\phi_n) * S\phi_n(x) - ay_N(x,\varepsilon,f), \Rightarrow \varepsilon y'_N(x,\varepsilon,f)$$

$$+ay_N(x,\varepsilon,f) = \sum_{n=1}^{N} (f,S\phi_n) * S\phi_n(x),$$

Where $S\phi_n(x) = \phi_n(1-x), n = 1,2,...,$ Since, $B = \frac{a}{\varepsilon} > 0$, by Lemma 2.1, the system $\{\varphi_n(x)\}, n = 1,2,...,$ forms an orthonormal basis of the space $L^2(0.1)$.. The operator S is unitary, therefore it takes an orthonormal basis to an orthonormal basis, hence the system $\{S\varphi_n\}, n = 1,2,...,$ is also an orthonormal basis of the space $L^2(0,1)$.. Therefore, the function

$$f_N(\mathrm{x}) = \sum_{n=1}^{N} (\mathrm{f}, \mathrm{S}\varphi_n) \cdot \mathrm{S}\varphi_n(\mathrm{x})$$

is a partial sum of the Fourier series $f(x)$ of the function in the system $\{S\varphi_n\}, n = 1, 2, ...,$;

$$\lim_{N \to \infty} f_N(x) = f(x)$$

in space $L^2(0,1)$. Thus, we have proved the limiting relation

$$L_\varepsilon y_N = \varepsilon y_N'(x, \varepsilon, f) + a y_N(x, \varepsilon, f) = f_N(x) \to f(x), \text{ for } N \to \infty.$$

Now we show that the sequence $\{y_N\}, N = 1, 2...$ also converges in $L^2(0,1)$, for this it is sufficient to show its fundamental property in $L^2(0,1)$,

$$\|y_{N'} - y_{N''}\|^2 = \frac{1}{a^2} \sum_{N'}^{N''} (st, \varphi_n) \cos\sqrt{\mu n} \varphi_n(x) \|^2 = \frac{1}{a^2} \cdot \sum_{N'}^{N''} |(st, \varphi_n|^2 \cos\sqrt{\mu n} \le \frac{1}{a^2} \cdot \sum_{N'}^{N''} |(st, \varphi_{n})|^2 < \varepsilon, \forall N', N'' > N(\varepsilon)$$

since, the Parseval equality holds

$$\sum_{n=1}^{\infty} |(Sf, \varphi_n)|^2 = \|(Sf\|^2 = (Sf, Sf) = (S^2 f, f) = \|f\|^2 < +\infty$$

By definition, the function $y(x, \varepsilon, f) = \lim_{N \to \infty} y_N(x, \varepsilon, f)$ is a strong solution of the singularly perturbed Cauchy problem (1.1) - (1.2) in the space $L^2(0,1)$.

Corollary 3.1. If $a > 0$, then inequality

$$\|L_\varepsilon^{-1}\| \le \frac{1}{a^2}. \tag{3.6}$$

Evidence.

It follows from (3.3) that

$$\|y(x, \varepsilon, f)\|^2 = \|L_\varepsilon^{-1} f(x)\|^2 = \frac{1}{a^2} \cdot \sum_{n=1}^{\infty} |(Sf, \varphi_{n})|^2 \cos^2 \sqrt{\mu_n} \le \left| \mu_n > (\frac{\pi}{2})^2, \sqrt{\mu_n} - \text{вещ.} \right| \le \frac{1}{a^2} \sum_{n=1}^{\infty} |(Sf, \varphi_n|^2 =$$

$$= \frac{\|(sf\|^2}{a^2} = \frac{(st, sf)}{a^2} = \frac{\|f\|^2}{a^2}, \to \|y(x, \varepsilon, f)\| = \|L_\varepsilon^{-1} f\| \le \frac{\|f\|}{a}, \to \|L_\varepsilon^{-1}\| \le \frac{1}{a}$$

Theorem 3.2. If $a > 0$, and $f(x) \in W_2^n[0,1], n \ge 0,$ then the strong solution of the singularly perturbed Cauchy problem

$$\varepsilon y'(x) + a \cdot y(x) = f(x), x \in (0,1];$$
$$y(0) = 0$$

belongs to the space $W_2^{n+1}[0,1]$, and satisfies the estimate

$$\left\| y(x, \varepsilon, f) - \sum_{k=0}^{n-1} \frac{(-1)^k \left[f^{(k)}(x) - f^{(k)}(0) e^{-\frac{ax}{\varepsilon}} \right] \varepsilon^k}{a^{k+1}} \right\| \le \frac{\varepsilon^n}{a^{k+1}} \|f^{(n)}(x)\|.$$

Evidence.

By Theorem 3.1, the strong solution of problem (3.1) - (3.2) has the form

$$y(x,\varepsilon,f)=-\frac{1}{a}\sum_{n=1}^{\infty}(\mathrm{S}f,\varphi_n)\cos\sqrt{\mu_n}\varphi_n(x),$$

where the Fourier coefficients of the function in the system We calculate these coefficients.

$$(Sf,\varphi_n)=(f,\frac{\varphi_n^{'}(x)}{b}-\frac{\varphi_n(1-x)}{\cos\sqrt{\mu n}})=-\frac{(\mathrm{S}f,\varphi_n^{'})}{b}-\frac{(\mathrm{S}f,\mathrm{S}\varphi_n)}{\cos\sqrt{\mu n}}=-\frac{(\mathrm{S}f,\varphi_n^{'})}{b}-\frac{(f,\varphi_n)}{\cos\sqrt{\mu n}}$$

Where $b=a/\varepsilon>0$

$$(\mathrm{S}f,\varphi_n^{'})=\int_0^1 Sfd\varphi_n=Sf\cdot\varphi_n(x)\int_0^1-\int_0^1(\mathrm{S}f)^{'}\varphi_n(x)dx=f(0)\varphi_n(1)-((Sf)^{'},\varphi_n),$$

$$(\mathrm{S}f,\varphi_n)=-\frac{f(0)\varphi_n(1)}{b}+\frac{((\mathrm{S}f),\varphi_n^{'})}{b}-\frac{(f,\varphi_n)}{\cos\sqrt{\mu_n}};$$

Therefore

$$\left(Sf,\varphi_n\right)\cos\sqrt{\mu_n}=-f(0)\frac{\varphi_n(1)}{b}\cos\sqrt{\mu_n}-(f,\varphi_n)+\frac{((\mathrm{S}f)',\varphi_n)}{b}\cos\sqrt{\mu_n}$$

Substituting the obtained formula, in Fourier the representation of the solution $y(x,\varepsilon,f)$, and using Lemma 2.3, we have

$$y(x,\varepsilon,f)=-\frac{1}{a}\sum_{n=1}^{\infty}(sf,\phi_n)\cos\sqrt{\mu_n}\cdot\phi_n(x)=$$

$$=-\frac{1}{a}\cdot\sum_{n=1}^{\infty}\left[f(0)\frac{\phi_n(1)}{b}\cos\sqrt{\mu_n}-(f,\phi_n+\frac{((sf)',\phi_n)}{b}\cos\sqrt{\mu_n}\right]\cdot\phi_n(x)=$$

$$=\frac{f(0)}{a}\cdot\sum_{n=1}^{\infty}\frac{\phi_n(1)}{b}\cos\sqrt{\mu_n}\cdot\phi_n(x)+\frac{1}{a}\sum_{n=1}^{\infty}(f,\phi_n)\phi_n(x)-\frac{1}{a\cdot b}\cdot\sum_{n=1}^{\infty}((sf)',\phi_n)\cos\sqrt{\mu_n}\cdot\phi_n(x)=$$

$$=\frac{f(x)}{a}-\frac{f(0)}{a}\cdot\sum_{n=1}^{\infty}(e^{-bx},\phi_n)\cdot\phi_n(x)-\frac{1}{ab}\sum_{n=1}^{\infty}((sf)',\phi_n)\cos\sqrt{\mu_n}\cdot\phi_n(x)=$$

$$=\frac{f(x)}{a}-\frac{f(0)}{a}e^{-bx}+\frac{1}{ab}\cdot\sum_{n=1}^{\infty}(sf',\phi_n)\cos\sqrt{\mu_n}\cdot\phi_n(x)=$$

$$=\frac{f(x)}{a}-\frac{f(0)}{a}e^{-bx}-\frac{1}{b}\left[-\frac{1}{a}\sum_{n=1}^{\infty}(sf',\phi_n)\cos\sqrt{\mu_n}\cdot\phi_n(x)\right]=$$

$$=\frac{f(x)-f(0)e^{-\frac{a}{\varepsilon}x}}{a}-\frac{\varepsilon}{a}\cdot y(x,\varepsilon,f') \tag{3.7}$$

Replacing, in this formula f by f', we get

$$y(x,\varepsilon,f')=\frac{f'(x)}{a}-\frac{f'(0)}{a}e^{-\frac{a}{\varepsilon}x}-\frac{\varepsilon}{a}y(x,\varepsilon,f'')$$

Consequently,

$$y(x,\varepsilon,f)=\frac{f(x)}{a}-\frac{f(0)}{a}e^{-\frac{a}{\varepsilon}x}-\frac{\varepsilon}{a}\left[\frac{f'(x)}{a}-\frac{f'(0)}{a}e^{-\frac{a}{\varepsilon}x}-\frac{\varepsilon}{a}y(x,\varepsilon,f')\right]=$$

$$=\frac{f(x)}{a}-\frac{f(0)}{a}e^{-\frac{a}{\varepsilon}x}-\frac{\varepsilon}{a}\left[\frac{f'(x)}{a}-\frac{f'(0)}{a}e^{-\frac{a}{\varepsilon}x}-\frac{\varepsilon}{a}y(x,\varepsilon,f'')\right]=$$

$$=\frac{f(x)}{a}-\frac{f(0)}{a}e^{-\frac{a}{\varepsilon}x}-\left[f'(x)-f'(0)\cdot e^{-\frac{a}{\varepsilon}x}\right]\cdot\frac{\varepsilon}{a^2}+\frac{\varepsilon^2}{a^2}y(x,\varepsilon,f'')=$$

$$=\frac{f(x)-f(0)\cdot e^{-\frac{a}{\varepsilon}x}}{a}-\left[f'(x)-f'(0)\cdot e^{-\frac{a}{\varepsilon}x}\right]\cdot\frac{\varepsilon}{a^2}+\frac{\varepsilon^2}{a^2}y(x,\varepsilon,f'')$$

Suppose that the formula

$$y(x,\varepsilon,f)=\sum_{k=0}^{n-1}\frac{(-1)^k\left[f^{(k)}(x)-f^{(k)}(0)\cdot e^{-\frac{a}{\varepsilon}x}\right]\varepsilon^k}{a^{k+1}}+(-1)^n\frac{\varepsilon^n}{a^n}y(x,\varepsilon,f^{(n)}) \quad (3.8)$$

is true. Then, by virtue of formula (3.7), we have

$$y(x,\varepsilon,f^k)=\frac{f^n(x)}{a}-\frac{f_n(0)}{a}\cdot e^{-\frac{a}{\varepsilon}x}-\frac{\varepsilon}{a}y(x,\varepsilon,f^{(n+1)}) \ ,$$

Consequently,

$$y(x,\varepsilon,f)=\sum_{k=0}^{n-1}\frac{(-1)^k\left[f^{(k)}(x)-f^{(k)}(0)\cdot e^{-\frac{a}{\varepsilon}x}\right]\varepsilon^k}{a^{k+1}}+(-1)^n\frac{\varepsilon^n}{a^n}\left[\frac{f^{(n)}(x)-f^{(n)}(0)\cdot e^{-\frac{a}{\varepsilon}x}}{a}-\frac{\varepsilon}{a}y(x,\varepsilon,f^{(n+1)})\right]=$$

$$=\sum_{k=0}^{n}\frac{(-1)^k\left[f^{(k)}(x)-f^{(k)}(0)\cdot e^{-\frac{a}{\varepsilon}x}\right]\varepsilon^k}{a^{k+1}}+(-1)^{n+1}\frac{\varepsilon^{n+1}}{a^{n+1}}y(x,\varepsilon,f^{(n+1)}).$$

Thus, the validity of (3.8) is proved, from which the theorem follows:

$$\left\|y(x,\varepsilon,f)-\sum_{R=0}^{n-1}\frac{(-1)^R\left[f^{(R)}(x)-f^{(R)}(0)\cdot e^{-\frac{a}{\varepsilon}x}\right]\varepsilon^R}{a^{R+1}}\right\|^2=\frac{\varepsilon^{2n}}{a^{2n}}\left\|y\left(x,\varepsilon,f^{(n)}\right)\right\|^2=$$

$$=\frac{\varepsilon^{2n}}{a^{2n}}\cdot\frac{1}{a^2}\cdot\sum_{m=1}^{+\infty}\left|\left(sf^{(n)},\varphi_m\right)\right|^2\cos^2\sqrt{\mu_m}\le\frac{\varepsilon^{2n}}{a^{2n+2}}\cdot\sum_{m=1}^{\infty}\left|\left(sf^{(n)},\varphi_m\right)\right|^2\le\frac{\varepsilon^{2n}}{a^{2n+2}}\left\|sf^{(n)}\right\|^2\le\frac{\varepsilon^{2n}\left\|f^{(n)}\right\|^2}{a^{2n+2}},$$

those.

$$\left\| y(x,\varepsilon,f) - \sum_{R=0}^{n-1} \frac{(-1)^R \left[f^{(R)}(x) - f^{(R)}(0)\cdot e^{-\frac{a}{\varepsilon}x} \right] \varepsilon^R}{a^{R+1}} \right\| \le \frac{\varepsilon^n}{a^{n+1}} \cdot \left\| f^{(n)}(x) \right\|.$$

Theorem 3.3. If a the real value is nonzero, then the sequence of operators L_ε^{-1} converges strongly to the operator $L_0^{-1} = I/a$, for $\varepsilon \to +0$, if and only if $a > 0$.

Evidence.

A) Necessity. Let $f(x)$ an arbitrary absolutely continuous function satisfying the condition $f(0) \ne 0$, then from formula (3.7), we have

$$L_\varepsilon^{-1} f(x) - L_0^{-1} f(x) = -\frac{f(0)}{a} \cdot e^{-\frac{a}{\varepsilon}x} - \frac{\varepsilon}{a} L_\varepsilon^{-1} f'(x), \Rightarrow \frac{|f(0)|}{a} \cdot \left\| e^{-\frac{a}{\varepsilon}x} \right\| \le \left\| L_\varepsilon^{-1} f - L_0^{-1} f \right\| \le \frac{\varepsilon}{|a|} \left\| L_\varepsilon^{-1} f'(x) \right\|, \Rightarrow$$

$$\frac{|f(0)|}{|a|} \cdot \frac{\left| \left(1 - e^{-\frac{2a}{\varepsilon}} \right) \right| \varepsilon}{2|a|} \le \left\| L_\varepsilon^{-1} f - L_0^{-1} f \right\| + \frac{\varepsilon}{|a|} \left\| L_\varepsilon^{-1} f' \right\|;$$

If $a < 0$, then
$$\left| \left(1 - e^{-\frac{2a}{\varepsilon}} \right) \right| = e^{-\frac{2a}{\varepsilon}} - 1 > \frac{1}{2} \left(\frac{2a}{\varepsilon} \right)^2 \ge \frac{2a^2}{\varepsilon^2}, \Rightarrow$$

$$\frac{\left| \left(1 - e^{-\frac{2a}{\varepsilon}} \right) \right| \varepsilon}{2a^2} > \frac{1}{\varepsilon}, \Rightarrow \frac{|f(0)|}{\varepsilon} \le \left\| L_\varepsilon^{-1} f - L_0^{-1} f \right\| + \frac{\varepsilon}{|a|} \left\| L_\varepsilon^{-1} f' \right\|; \tag{3.9}$$

Suppose that strong convergence holds, when $L_\varepsilon^{-1} \to L_0^{-1}$, then $\varepsilon \to +0$ the left-hand side of (3.9) tends to $+\infty$, and the right-hand side tends to zero, which is impossible, hence, if strong convergence holds $L_\varepsilon^{-1} \to L_0^{-1}$, then certainly $a > 0$.

B) Sufficiency. Let $a > 0$, then, by virtue of inequality (3.6), we have $\left\| L_\varepsilon^{-1} \right\| \le \frac{1}{a} \forall \varepsilon > 0$

Assuming $f = S\varphi_m$, from the formula (3.3), we have $L_\varepsilon^{-1} S\varphi_m = -\frac{1}{a} \cos\sqrt{\mu_m} \cdot \varphi_m(x), m = 1, 2, ...$

If $\varepsilon \to +0$, then $b = a/\varepsilon \to +\infty$, , by Lemma 2.1, we have

$$\sqrt{\mu_m} \to m\pi, \left\| \varphi_m(x) - \sqrt{2} \sin m\pi x \right\| \to 0, \text{ at } \varepsilon \to +0 .$$

Then $-\frac{1}{a} \cos\sqrt{\mu_m} \cdot \varphi_m(x) \to \frac{(-1)^{m+1}}{a} \sqrt{2} \sin m\pi x = \frac{\sqrt{2} \sin m\pi (1-x)}{a} = \frac{S}{a} \sqrt{2} \sin m\pi x.$

Assuming, for convenience $\varphi_m^0(x) = \sqrt{2} \sin m\pi x, m = 1, 2, ...$, we have

$$\left\| L_\varepsilon^{-1} S\varphi_m^0 - L_0^{-1} S\varphi_m^0 \right\| \le \left\| L_\varepsilon^{-1} S\varphi_m^0 - L_\varepsilon^{-1} S\varphi_m \right\| + \left\| L_\varepsilon^{-1} S\varphi_m - L_0^{-1} S\varphi_m^0 \right\| \le \left\| L_\varepsilon^{-1} \right\| \cdot \left\| S\varphi_m^0 - S\varphi_m \right\| + \left\| L_\varepsilon^{-1} \varphi_m - L_0^{-1} S\varphi_m^0 \right\| \le$$

$$\le \frac{\left\| \varphi_m^0 - \varphi_m \right\|}{a} + \left\| L_\varepsilon^{-1} S\varphi_m - \frac{S}{a} \varphi_m^0 \right\| \to 0 \text{ at } \varepsilon \to +0 .$$

Therefore $\left\|L_\varepsilon^{-1}S\varphi_m^0 - L_0^{-1}S\varphi_m^0\right\| \to 0$, when $\varepsilon \to +0$. The system $\varphi_m^0(x) = \sqrt{2}\sin m\pi x, m = 1,2,...$ forms an orthonormal basis of space $L^2(0,1)$, therefore the system $\{S\varphi_m^0\}, m = 1,2,...$ is also the basis of this space. Consequently, the linear span of these vectors $\{S\varphi_m^0\}, m = 1,2,...$ forms a dense set in space $L^2(0,1)$. Then the assertion of the theorem follows from the Banach-Steinhaus theorem.

Remark 3.1. Formula (3.7) is true for any complex value a with nonzero real part.

Proof 1. In fact, the solution of the Cauchy problem

$$y'(x) + a\cdot y(x) = f(x), x \in (0,1]$$
$$y(0) = 0$$

has the form

$$y(x,\varepsilon,f) = \frac{1}{\varepsilon}\int_0^x f(t)e^{-\frac{a}{\varepsilon}(x-t)}dt, \tag{3.10}$$

therefore the following chain of equalities holds

$$y(x,\varepsilon,f') = \frac{1}{\varepsilon}\int_0^x e^{-\frac{a}{\varepsilon}(x-t)}f'(t)dt = \frac{1}{\varepsilon}\int_0^x e^{-\frac{a}{\varepsilon}(x-t)}df = \frac{f(t)e^{-\frac{a}{\varepsilon}(x-t)}}{\varepsilon}\Big|_0^x - \frac{1}{\varepsilon}\int_0^x f(t)e^{-\frac{a}{\varepsilon}(x-t)}\cdot\frac{a}{\varepsilon} =$$

$$= \frac{f(x) - f(0)e^{-\frac{a}{\varepsilon}x}}{\varepsilon} - \frac{a}{\varepsilon}y(x,\varepsilon,f), \Rightarrow \varepsilon\,\mathrm{y}(x,\varepsilon,f') = f(x) - f(0)e^{-\frac{a}{\varepsilon}x} - ay(x,\varepsilon,f), \Rightarrow ay(x,\varepsilon,f) =$$

$$f(\mathrm{x}) - \mathrm{f}(0)e^{-\frac{a}{\varepsilon}x} - \varepsilon y(x,\varepsilon,f'), y(x,\varepsilon,f) = \frac{f(x)}{a} - \frac{f(0)}{a}e^{-\frac{a}{\varepsilon}x} - \frac{\varepsilon}{a}\cdot y(x,\varepsilon,f').$$

Proof 2. By Lemma 2.4, the operators $A_\varepsilon y = \varepsilon y'(x) + a\cdot y(x); y(0) = 0$ and $B_\varepsilon z = \varepsilon z'(x) + a\cdot z(x); \mathrm{z}(0) = 0$ are similar to each other, where the similarity operator is

$$V_\varepsilon f(x) = e^{-i\frac{jma}{\varepsilon}x}\cdot f(x)$$

The formula (3.7) is valid for real numbers a, and therefore

$$B_\varepsilon^{-1}f = \frac{f(x)}{\mathrm{Re}a} - \frac{f(0)}{\mathrm{Re}a}e^{-\frac{\mathrm{Re}a}{\varepsilon}x} - \frac{\varepsilon}{\mathrm{Re}a}B_\varepsilon^{-1}(f'), \Rightarrow \mathrm{Re}aB_\varepsilon^{-1}f = f(x) - f(0)e^{-\frac{\mathrm{Re}a}{\varepsilon}x} - \varepsilon B_\varepsilon^{-1}(f').$$

By virtue of similarity, the following formula holds: $V_\varepsilon^{-1}A_\varepsilon^{-1} = B_\varepsilon^{-1}V_\varepsilon^{-1}$. If $f = V_\varepsilon^{-1}\varphi$, where ε is fixed, then

$$\operatorname{Re} a V_\varepsilon^{-1} A_\varepsilon^{-1}\varphi = V_\varepsilon^{-1}\varphi - \varphi(0) e^{-\frac{\operatorname{Re} a}{\varepsilon}x} - \varepsilon B_\varepsilon^{-1}\left(\frac{d}{dx}V_\varepsilon^{-1}\varphi\right);$$

$$\frac{d}{dx}V_\varepsilon^{-1}\varphi = \frac{d}{dx}e^{-i\frac{jma}{\varepsilon}x}\varphi = i\frac{jma}{\varepsilon}V_\varepsilon^{-1}\varphi + V_\varepsilon^{-1}\varphi', \Rightarrow$$

$$\operatorname{Re} a V_\varepsilon^{-1} A_\varepsilon^{-1}\varphi = V_\varepsilon^{-1}\varphi - \varphi(0) e^{-\frac{\operatorname{Re} a}{\varepsilon}x} - ijmaB_\varepsilon^{-1}V_\varepsilon^{-1}\varphi - \varepsilon B_\varepsilon^{-1}V_\varepsilon^{-1}\varphi', \Rightarrow$$

$$aV_\varepsilon^{-1} A_\varepsilon^{-1}\varphi = V_\varepsilon^{-1}\varphi - \varphi(0) e^{-\frac{\operatorname{Re} a}{\varepsilon}x} - \varepsilon V_\varepsilon^{-1} A_\varepsilon^{-1}\varphi', \Rightarrow$$

$$aA_\varepsilon^{-1}\varphi = \varphi - \varphi(0)\cdot e^{-\frac{\operatorname{Re} a}{\varepsilon}x} - \varepsilon A_\varepsilon^{-1}\varphi', \Rightarrow$$

$$A_\varepsilon^{-1}\varphi = \frac{\varphi(x)}{a} - \frac{\varphi(0)}{a}\cdot e^{-\frac{a}{\varepsilon}x} - \frac{\varepsilon}{a}A_\varepsilon^{-1}\varphi';$$

Hence, $y(x,\varepsilon,\varphi) = \dfrac{\varphi(x)}{a} - \dfrac{\varphi(0)}{a}e^{-\frac{a}{\varepsilon}x} - \dfrac{\varepsilon}{a}y(x,\varepsilon,\varphi')$, as required.

Proof 3. Integrating by parts the right-hand side of (3.10), we have

$$y(x,\varepsilon,f) = \frac{1}{\varepsilon}\int_0^x f(t)e^{-\frac{a}{\varepsilon}(x-t)}dt = \frac{1}{a}\int_0^x f(t)\,de^{-\frac{a}{\varepsilon}(x-t)} =$$

$$\frac{f(t)e^{-\frac{a}{\varepsilon}(x-t)}}{a}\Big|_0^x - \frac{1}{a}\int_0^x f'(t)e^{-\frac{a}{\varepsilon}(x-t)}dt = \frac{f(x)-f(0)e^{-\frac{a}{\varepsilon}x}}{a}\cdot\frac{\varepsilon}{a}y(x,\varepsilon,f').$$

Theorem 3.4. A sequence of operators L_ε^{-1} strongly converges to an operator L_0^{-1}, for $\varepsilon \to +0$, in a space $H = L^2(0,1)$ if and only if the following inequality holds:

$$\operatorname{Re} a > 0 \tag{3.11}$$

Evidence.

A) **Necessity**. Let the sequence of operators L_ε^{-1} strongly converge to the operator L_0^{-1}. It is not difficult to see that the function $z(x,\varepsilon) = e^{-\frac{a}{\varepsilon}x} - 1$ is a solution of the Cauchy problem $\varepsilon z'(x) + a\cdot z(x) = -a; z(0) = 0,$, so we have formulas

$$z(x,\varepsilon) = L_\varepsilon^{-1}(a) = -aL_\varepsilon^{-1}(1), \Rightarrow e^{-\frac{a}{\varepsilon}x} = 1 - aL_\varepsilon^{-1}(1), \Rightarrow \left\| e^{-\frac{a}{\varepsilon}x}\right\| \le 1 + |a|\left\|L_\varepsilon^{-1}\right\|;$$

Let us calculate the left-hand side of this inequality:

$$\left\| e^{-\frac{a}{\varepsilon}x}\right\|^2 = \int_0^1 e^{-\frac{a}{\varepsilon}x} - e^{-\frac{\bar a}{\varepsilon}x}dx = \int_0^1 e^{-\frac{2\operatorname{Re} a}{\varepsilon}}dx = -\frac{\varepsilon}{2\operatorname{Re} a}e^{-\frac{2\operatorname{Re} a}{\varepsilon}x}\Big|_0^1 = \frac{\varepsilon}{2\operatorname{Re} a}\left(1 - e^{-\frac{2\operatorname{Re} a}{\varepsilon}}\right).$$

Consequently, for any a, with $\operatorname{Re} a \ne 0$, the following inequality holds

$$\sqrt{\frac{\varepsilon}{2\operatorname{Re} a}\left(1 - e^{-\frac{2\operatorname{Re} a}{\varepsilon}}\right)} \le 1 + |a|\left\|L_\varepsilon^{-1}\right\|.$$

If $\operatorname{Re} a < 0$, then $\left|1-e^{-\frac{2\operatorname{Re} a}{\varepsilon}}\right| = e^{-\frac{2\operatorname{Re} a}{\varepsilon}} - 1 > \frac{1}{2}\left(-\frac{2\operatorname{Re} a}{\varepsilon}\right)^2 = \frac{2(\operatorname{Re} a)^2}{\varepsilon^2}$, therefore

$$\frac{\varepsilon}{2}\left|\frac{1-e^{-\frac{2\operatorname{Re} a}{\varepsilon}}}{\operatorname{Re} a}\right| > \frac{\varepsilon}{2}\frac{2(\operatorname{Re} a)^2}{\varepsilon^2|\operatorname{Re} a|} \geq \frac{|\operatorname{Re} a|}{\varepsilon} \text{ and } \frac{|\operatorname{Re} a|}{\varepsilon} \leq 1 + |a|\left\|L_\varepsilon^{-1}\right\|.$$

Therefore, $\lim_{\varepsilon \to +0}\left\|L_\varepsilon^{-1}\right\| = +\infty$ and this circumstance contradicts the Banach-Steinhaus theorem, since, according to this theorem, the sequence of norms $\left\|L_\varepsilon^{-1}\right\|$ is uniformly bounded. Thus, if $\operatorname{Re} a \neq 0$,, and strong convergence takes place $L_\varepsilon^{-1} \to L_0^{-1}$, then certainly $\operatorname{Re} a > 0$.

If $\operatorname{Re} a = 0$, then from Lemma 2.2, we have $\left\|L_\varepsilon^{-1}\right\| = \left\|A_\varepsilon^{-1}\right\| = \left\|B_\varepsilon^{-1}\right\|$ where $B^{-1} f(x) = \frac{1}{\varepsilon}\int_0^x f(t)\,dt$.

Then, by the Hardy inequality [20], we therefore have $\left\|L_\varepsilon^{-1}\right\| = \left\|A_\varepsilon^{-1}\right\| = \left\|B_\varepsilon^{-1}\right\| = \frac{1}{\varepsilon}\cdot\frac{2}{\pi}$, in this case $\lim_{\varepsilon \to +0}\left\|L_\varepsilon^{-1}\right\| = +\infty$, which contradicts the Banach-Steinhaus theorem.

Thus, if the sequence of operators L_ε^{-1} converges strongly to an operator I/a, then the inequality holds $\operatorname{Re} a > 0$.

Proof. Let $\operatorname{Re} a > 0$, and $f(x)$ an arbitrary function in the class $C_0^\infty(0,1)$, then, by (3.7), we have
$$y(x,\varepsilon,\mathrm{f}) = \frac{f(x)}{a} - \frac{f(0)}{a}e^{-\frac{a}{\varepsilon}x} - \frac{\varepsilon}{a}y(x,\varepsilon,f'),$$

and since $f(0) = 0$, then $\left\|y(x,\varepsilon,f) - \frac{f(x)}{a}\right\| \leq \frac{\varepsilon}{|a|}\left\|y(x,\varepsilon,f')\right\| \leq \frac{\varepsilon}{|a|}\frac{\left\|f'(x)\right\|}{\operatorname{Re} a} \to 0$

at $\varepsilon \to +0$, and this means that $\left\|L_\varepsilon^{-1} f - L_0^{-1}\right\| \to 0$, with $\varepsilon \to +0$, and $\operatorname{Re} a > 0$ for any $f(x) \in C_0^\infty(0,1)$.

By Lemma 2.2, and Corollary 3.1, we have $\left\|L_\varepsilon^{-1}\right\| \leq \frac{1}{\operatorname{Re} a}$.

Consequently, the assertion of Theorem 3.4 follows from the Banach-Steinhaus theorem, since linear manifolds of functions from the class are everywhere dense in space $H = L^2(0,1)$.

Theorem 3.5. If $\operatorname{Re} a = 0$, $a \neq 0$ then, the sequence of operators L_ε^{-1} strongly converges to an L_0^{-1} operator, for $\varepsilon \to +0$, in a subset $DCW_2^1(0,1)$, if and only if

$$D = \left\{f(x) \in W_2^1(0,1), f(0) = 0\right\}$$

Evidence. Let the sequence of operators L_ε^{-1} strongly converge to an L_0^{-1} operator, for $\varepsilon \to +0$, in some subset $DCW_2^1(0,1)$. Then from the representation

$y(x,\varepsilon,\mathrm{f}) = \dfrac{f(x)-f(0)e^{-\frac{a}{\varepsilon}x}}{a} - \dfrac{1}{a}\int_0^x f'(t)e^{-\frac{a}{\varepsilon}(x-t)}dt,$ fair for any $f(x)\in D$, we have $\dfrac{|f(0)|}{|a|} \le \left\|\dfrac{f(x)}{a} - L_\varepsilon^{-1}f\right\| + \dfrac{1}{|a|}\left\|\int_0^x f'(t)e^{-\frac{a}{\varepsilon}(x-t)}dt\right\|.$

The first term on the right-hand side of this inequality tends to zero when $\varepsilon \to +0$, by virtue of the Riemann-Lebesgue lemma, and Lebesgue's theorem on bounded convergence $f(0)=0$, and consequently the necessity of this condition is proved.

Conversely, if $f(x)\in D$ i.e. $f(x)\in W_2^1(0,1)$ and $f(0)=0$, in view of the same representation, we have $\left\|L_\varepsilon^{-1}f - \dfrac{f(x)}{a}\right\| = \dfrac{1}{|a|}\left\|\int_0^x f'(t)e^{-\frac{a}{\varepsilon}(\mathrm{x-t})}dt\right\|.$

The right-hand side of this formula tends to zero, when $\varepsilon \to +0$, by virtue of the Riemann-Lebesgue lemma, and Lebesgue's theorems on bounded convergence.

Theorem 3.6.If $\operatorname{Re} a = 0$, then the sequence of operators L_ε^{-1} is weakly convergent to the operator L_0^{-1}, for $\varepsilon \to +0$, in the subset $W_2^1(0,1)\subset L^2(0,1)$.

Evidence.

From the representation (3.7), we have $\left(L_\varepsilon^{-1}f - L_0^{-1}f\right) = \dfrac{-f(0)}{a}e^{\frac{-a}{\varepsilon}x} - \dfrac{1}{a}\int_0^x f'(t)e^{\frac{-a}{\varepsilon}(x-t)}dt$.

By the Riemann-Lebesgue lemma, the first term on the right-hand side of this formula converges weakly to zero for $\varepsilon \to +0$, and the second term strongly converges to zero in the subset $W_2^1(0,1)CL^2(0,1)$, by the same Riemann-Lebesgue lemma and Lebesgue's theorem on bounded convergence.

In connection with this theorem, the question arises: is it possible for $\operatorname{Re} a = 0$ weak convergence in the whole space $H = L^2(0,1)$?

Suppose that a sequence of operators L_ε^{-1} converges weakly to an operator L_0^{-1}, for $\varepsilon \to +0$, in the whole space $H = L^2(0,1)$, that is, for any $f\in L^2(0,1)$, the sequence of elements $\left\{L_\varepsilon^{-1}f\right\}$ converges weakly to an element $L_0^{-1}f$. Then, by the criterion for weak convergence (see, for example, [18], p.185), the sequence of norms $\left\{\left\|L_\varepsilon^{-1}f\right\|\right\}$ is bounded $\left\{L_\varepsilon^{-1}f\right\}$; The sequence is bounded for each fixed f of $H = L^2(0,1)$. Hence, by the principle of uniform boundedness, the sequence is bounded, and this is impossible $\left\{\left\|L_\varepsilon^{-1}\right\|\right\}$, since, for $\operatorname{Re} a = 0$, there is a formula $\left\|L_\varepsilon^{-1}\right\| = \dfrac{1}{\varepsilon} - \dfrac{2}{\pi}$ that is incompatible with the assertion obtained.

4. Discussion

1) If $\operatorname{Re} a > 0$, then L_ε^{-1} converges strongly to L_0^{-1} in H.

2) If $\operatorname{Re} a \le 0$, then there is not even a weak convergence in H.

3) If $\operatorname{Re} a = 0$ and $f\in W_2^1(0,1)$, then for $f(0)=0$ a sequence $\left\{L_\varepsilon^{-1}f\right\}$ converges to L_0^{-1} strongly, but for $f(0)\ne 0$ converges weakly.

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КРИТЕРИИ СИЛЬНОЙ СХОДИМОСТИ РЕШЕНИЙ СИНГУЛЯРНО ВОЗМУЩЕННОЙ ЗАДАЧИ КОШИ

Аннотация: Методом отклоняющегося аргумента решено интегральное уравнение первого порядка с негладкой правой частю. Показано,что решение соответствующей сингулярно возмущенной задачи Коши сходится к решению невозмущенной задачи. При этом использованы критерии сильной сходимости и спектральные свойства вспомогательной задачи.Найден корень квадратный от одного класса операторов Штурма-Лиувилля.

Ключевые слова: сильная сходимость, спектр, спектральное разложение, квадратный корень от оператора, уравнение с отклоняющимся аргументом, теорема Гильберта-Шмидта.

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СИНГУЛЯР ӘСЕРЛЕНГЕН КОШИ ЕСЕБІНІҢ ӘЛДІ ЖЫЙНЫҚТАЛУЫНЫҢ КЕПІЛДІГІ

Аннотация: Аргументін ауытқыту әдісімен бос мүшесі кедір-бұдыр бірінші ретті интегралдық теңдеу шешілген. Бұл есепке сәйкес сингуляр әсерленген Кошидің есебінің шешімі бастапқы теңдеудің шешіміне күшті жыйнақталатыны көрсетілді.Бұл үшін күшті жыйнақталудың үзілді кесілді белгісі мен көмекші есептің спектрәлдік қасиеттері қолданыды.Жолшыбай Штурм-Лиувилл операторының бір түрінің квадрат түбірі табылды.

Түйін сөздер: әлді жыйнақталу, спектрәлді таралым, оператордың квадрат түбірі, аргументі ауытқыған теңдеу, Гилберт-Шмидтің теоремасы.

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