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ҚАЗАҚ ҰЛТТЫҚ УНИВЕРСИТЕТІНІҢ

# Х А Б А Р Л А Р Ы

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НАЦИОНАЛЬНОЙ АКАДЕМИИ НАУК  
РЕСПУБЛИКИ КАЗАХСТАН

КАЗАХСКИЙ НАЦИОНАЛЬНЫЙ  
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## NEWS

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## STEFAN PROBLEM IN ELLIPSOIDAL COORDINATES

**Abstract.** This paper presents the quasi-stationary Stefan problem in symmetric electrical contacts. The method of the solution can be obtained from the suggestion that the identity of equipotential and isothermal surfaces in contacts, which is correct for stationary fields in linear case, keeps safe for non-linear case as well. The idea is, transform the system of problem which is given in cylindrical coordinates into ellipsoidal coordinates. The analytical solution of stationary Stefan problem is found. Based on that decision was constructed the temperature profile to the approximate solution of heat problem with Joule heating in ellipsoidal coordinates.

**Keywords:** quasi-stationary model, Stefan problem, integral method.

### Introduction

Stationary temperature and electromagnetic fields in symmetric electrical contacts have been described in [1]. Working with the scale of a mile second range, we think that every time the stationary state manages to instantly achieve stationary. And therefore this solution is suitable for constructing a temperature profile of the quasi-stationary problem.

### Quasi-stationary nonlinear mathematical model of melting in ellipsoidal coordinates

The system of equations for the temperature  $T_i(r, z)$  and electrical potential  $\Phi_i(r, z)$  can be written in the form

$$\operatorname{div}(\lambda_1 \operatorname{grad} T_1) + \frac{1}{\rho_1} \operatorname{grad}^2 \Phi_1 = 0$$

$$\operatorname{div}\left(\frac{1}{\rho_1} \operatorname{grad} \Phi_1\right) = 0$$

$$\operatorname{div}(\lambda_2 \operatorname{grad} T_2) + \frac{1}{\rho_2} \operatorname{grad}^2 \Phi_2 = 0$$

$$\operatorname{div}\left(\frac{1}{\rho_2} \operatorname{grad} \Phi_2\right) = 0$$

where  $\Phi_i$ ,  $\lambda_i$ ,  $\rho_i$  are electrical potential, heat conductance and electrical resistivity respectively.

In cylindrical coordinates these equations can be written as

$$\rho_i \frac{d\lambda_i}{dT_i} \left[ \left( \frac{\partial T_i}{\partial r} \right)^2 + \left( \frac{\partial T_i}{\partial z} \right)^2 \right] + \rho_i \lambda_i \Delta T_i + \left( \frac{\partial \Phi_i}{\partial r} \right)^2 + \left( \frac{\partial \Phi_i}{\partial z} \right)^2 = 0 \quad (1)$$

$$\frac{1}{\rho_i} \Delta \Phi_i - \frac{d\rho_i}{dT} \frac{1}{\rho_i^2} \left( \frac{\partial T_i}{\partial r} \frac{\partial \Phi_i}{\partial r} + \frac{\partial T_i}{\partial z} \frac{\partial \Phi_i}{\partial z} \right) = 0 \quad (2)$$

The index  $i=1$  relates to the melted zone occupying the domain  $D_1(0 < z < \infty, r_0 < r < r_m(t))$ , and  $i=2$  corresponds to the solid zone in the domain  $D_2(0 < z < \infty, r_m(t) < r < \infty)$ .

It has to be mentioned that this problem is essentially non-linear due to temperature dependence of thermal conductivity  $\lambda_i = \lambda_i(T_i)$  and electrical conductivity  $\rho_i = \rho_i(T_i)$ . The method of the solution can be obtained from the suggestion that the identity of equipotential and isothermal surfaces in contacts, which is correct for stationary fields in linear case, keeps safe for non-linear case as well. In linear case these surfaces are ellipsoids of revolution.

Equations (1) and (2) can be transformed into ellipsoidal coordinates and using well known relations among cylindrical and elliptical coordinates, if we suggest similarly like above that

$$\Phi_i = \Phi_i(\xi), \quad T_i = T_i(\xi), \quad (3)$$

where

$$\xi = \sqrt{s + \sqrt{s^2 + 4r_0^2 z^2}}, \quad s = r^2 + z^2 - r_0^2$$

then the equations (1) and (2) should be replaced by the equation

$$\rho_i \frac{d\lambda_i}{dT_i} \cdot \left( \frac{dT_i}{d\xi} \right)^2 + \rho_i \lambda_i \frac{d^2 T_i}{d\xi^2} + \rho_i \lambda_i \frac{dT_i}{d\xi} \cdot \frac{2\xi}{r_0^2 + \xi^2} + \left( \frac{d\Phi_i}{d\xi} \right)^2 = 0 \quad (4)$$

$$\frac{d^2 \Phi_i}{d\xi^2} + \frac{2\xi}{r_0^2 + \xi^2} \cdot \frac{d\Phi_i}{d\xi} - \frac{1}{\rho_i} \frac{d\rho_i}{dT_i} \cdot \frac{dT_i}{d\xi} \cdot \frac{d\Phi_i}{d\xi} = 0 \quad (5)$$

$$D: 0 < r < \infty, 0 < z < \infty, z = 0, \cup 0 \leq r < r_0, 0 < \xi < \infty, 0 \leq \eta < r_0 \quad (6)$$

The boundary conditions are

$$z = 0 (\xi = 0) \quad \frac{dT_1}{d\xi} = 0 \quad (7) \quad \Phi_1|_{0 \leq r \leq r_0} = 0 \quad (8) \quad \frac{\partial \Phi_1}{\partial z} \Big|_{r < r_m(t)} = 0 \quad (9)$$

$$z = \sigma(r, t) (\xi = \xi_m(t)) \quad T_1 = T_2 = T_m \quad (10) \quad \Phi_1 = \Phi \quad (11)$$

$$\lambda_1 \frac{dT_1}{d\xi} = \lambda_2 \frac{dT_2}{d\xi} \quad (12) \quad \frac{1}{\rho_1} \frac{d\Phi_1}{d\xi} = \frac{1}{\rho_2} \frac{d\Phi_2}{d\xi} \quad (13)$$

$$z = \infty \text{ or } r = \infty (\xi = \infty) \quad T_2 = 0 \quad (14) \quad \Phi_2 = \frac{U_c}{2} \quad (15)$$

while the solution for electric potentials

$$\Phi_1'(\xi) = \frac{I^2 \rho_1(T_1)}{2\pi(r_0^2 + \xi^2)}, \quad \Phi_2'(\xi) = \frac{I^2 \rho_2(T_2)}{2\pi(r_0^2 + \xi^2)} \quad (16)$$

Putting (16) into (4) we get

$$\frac{1}{\lambda_i} \frac{d\lambda_i}{dT_i} \cdot \left( \frac{dT_i}{d\xi} \right)^2 + \frac{d^2 T_i}{d\xi^2} + \frac{dT_i}{d\xi} \cdot \frac{2\xi}{r_0^2 + \xi^2} + \frac{I^2 \rho_i}{4\pi^2(r_0^2 + \xi^2)} = 0 \quad (17)$$

Let us introduce the new independent variable  $\zeta$  using formula and consider the case when thermal conductivity doesn't depend on temperature,  $\frac{d\lambda}{dT} = 0$

$$\zeta = \arctan \frac{\xi}{r_0} \quad (18)$$

Taking into account that  $\rho_1 = \rho_{10}(1 + \alpha_{10}(T_1 - T_m))$ ,  $\rho_2 = \rho_{20}(1 + \alpha_{20}T_2)$

And using[2]

$$\omega_i^2 = \frac{I^2 \rho_{i0} \alpha_{i0}}{4\pi^2 r_0^2 \lambda_i}$$

then the equation (17) for melted zone can be reduced to the form

$$\frac{d^2 T_1}{d\zeta^2} + \frac{\omega_1^2}{\alpha_{10}} [1 + \alpha_{10}(T_1 - T_m)] = 0 \quad (19)$$

The general solution of this equation is

$$T_1 = \frac{A_1}{\alpha_{10}} \cos \omega_1 \zeta + \frac{B_1}{\alpha_{10}} \sin \omega_1 \zeta + T_m - \frac{1}{\alpha_{10}} \quad (20)$$

and  $A_1, B_1$  are arbitrary constants, which can be found from the boundary conditions (7) and (8)

From (7) and (10)

$$B_1 = 0$$

$$A_1 = \frac{1}{\cos \omega_1 \frac{\pi}{2}}$$

Finally,

$$T_1 = \frac{1}{\alpha_{10}} \left( \frac{\cos \omega_1 \zeta}{\cos \omega_1 \zeta_m} + \alpha_{10} T_m - 1 \right)$$

The equation (17) for solid zone can be reduced to the form

$$\frac{d^2 T_2}{d\zeta^2} + \frac{\omega_2^2}{\alpha_{20}} [1 + \alpha_{20} T_2] = 0 \quad (21)$$

the general solution can be represented

$$T_2 = \frac{1}{\alpha_{20}} \left( A_2 \frac{\cos \omega_2 \zeta}{\cos \omega_2 \frac{\pi}{2}} + B_2 \frac{\sin \omega_2 \zeta}{\sin \omega_2 \frac{\pi}{2}} - 1 \right) \quad (22)$$

From (14) and (10) can be found  $A_2, B_2$  and temperature  $T_2$  will be in the form

$$T_2 = \frac{1}{\alpha_{20} \sin \omega_2 \left( \frac{\pi}{2} - \zeta_m \right)} \left\{ (1 + \alpha_{20} T_m) \sin \omega_2 \left( \frac{\pi}{2} - \zeta \right) - \sin \omega_2 (\zeta_m - \zeta) - \sin \omega_2 \left( \frac{\pi}{2} - \zeta_m \right) \right\}$$

Noting first that

$$\frac{dT_1}{d\xi} \Big|_{\xi=\xi_m(t)} = \frac{dT_1}{d\zeta} \cdot \frac{d\zeta}{d\xi} \Big|_{\xi=\xi_m(t)} = -\frac{\omega_1 \sin \omega_1 \zeta_m}{\alpha_{10} \cos \omega_1 \zeta_m} \cdot \frac{r_0}{r_0^2 + \xi_m^2(t)}$$

$$\frac{dT_2}{d\xi} \Big|_{\xi=\xi_m(t)} = \frac{dT_2}{d\zeta} \cdot \frac{d\zeta}{d\xi} \Big|_{\xi=\xi_m(t)} = -\frac{\omega_2 \left[ (1 + \alpha_{20} T_m) \cos \left( \frac{\pi}{2} - \zeta_m \right) - 1 \right]}{\alpha_{20} \sin \omega_2 \left( \frac{\pi}{2} - \zeta_m \right)} \cdot \frac{r_0}{r_0^2 + \xi_m^2(t)}$$

from (12),

$$\frac{\lambda_1 \omega_1 \sin \omega_1 \zeta_m}{\alpha_{10} \cos \omega_1 \zeta_m} = \frac{\lambda_2 \omega_2 \left[ (1 + \alpha_{20} T_m) \cos \omega_2 \left( \frac{\pi}{2} - \zeta_m \right) - 1 \right]}{\alpha_{20} \sin \omega_2 \left( \frac{\pi}{2} - \zeta_m \right)}$$

finally we get

$$\zeta_m = \frac{1}{\omega_1} \arctan \frac{\lambda_2 \omega_2 \alpha_{10}}{\lambda_1 \omega_1 \alpha_{20}} \left[ (1 + \alpha_{20} T_m) \cot \omega_2 \left( \frac{\pi}{2} - \zeta_m \right) - \operatorname{cosec} \omega_2 \left( \frac{\pi}{2} - \zeta_m \right) \right]$$

#### Approximate solution of heat problem in ellipsoidal coordinates

Considering the problem from the class of Stefan type problem, in first stage of heating electrical contact, where contact material is solid and temperature attains softening point. In this case we consider the heat equation

$$\frac{\partial \theta_1}{\partial t} = \frac{a_1^2}{r_0^2} \cos^4(\zeta) \left[ \frac{\partial^2 \theta_1}{\partial \zeta^2} + \omega_1^2 \left( \theta_1 + \frac{1}{\alpha_1} \right) \right] \quad 0 < \zeta < \pi/2 \quad (23)$$

subjected to boundary conditions

$$\zeta = 0:$$

$$\frac{\partial \theta_1(0, t)}{\partial \zeta} = 0 \quad (24)$$

$$\zeta = \pi/2:$$

$$\theta_1(\pi/2, t) = 0 \quad (25)$$

and initial condition

$$t = 0:$$

$$\theta_1(\zeta, 0) = 0 \quad (26)$$

where

$$\omega_1 = \frac{I}{2\pi r_0^2} \sqrt{\frac{\rho_{10} \alpha_1}{c_1 \gamma_1}}$$

For the temperature distribution  $\theta_1(\zeta, t)$ , let us assume that the temperature profile as given in the form



$$\theta_1(\zeta, t) = A_1(t) \cos(\omega_1 \zeta) + B_1(t) \sin(\omega_1 \zeta) + C_1(t) \text{ in } 0 \leq \zeta \leq \frac{\pi}{2} \quad (27)$$

where the coefficients are in general functions of time.

Using conditions (24) and (25) we get

$$\begin{cases} B_1(t) = 0 \\ A_1(t) \cos(\omega_1 \frac{\pi}{2}) + C_1(t) = 0 \end{cases} \quad (28)$$

Integration equation (23) with respect to the space variable from  $\zeta = 0$  to  $\zeta = \pi/2$ , noting first that

$$\begin{aligned} \int_0^{\pi/2} \cos^4(\zeta) \left[ \frac{\partial^2 \theta_1}{\partial \zeta^2} + \omega_1^2 \left( \theta_1 + \frac{1}{\alpha_1} \right) \right] d\zeta &= \cos^4(\zeta) \frac{\partial \theta_1}{\partial \zeta} \Big|_0^{\pi/2} + 4\theta_1 \cos^3(\zeta) \sin(\zeta) \Big|_0^{\pi/2} + \\ &+ \frac{3\pi\omega_1^2}{16\alpha_1} + \int_0^{\pi/2} [12 \cos^2(\zeta) + (\omega_1^2 - 16) \cos^4(\zeta)] \theta_1 d\zeta \end{aligned}$$

then we have

$$\frac{r_0^2}{a_1^2} \int_0^{\pi/2} \frac{\partial \theta_1}{\partial t} d\zeta = \frac{3\pi\omega_1^2}{16\alpha_1} + \int_0^{\pi/2} [12 \cos^2(\zeta) + (\omega_1^2 - 16) \cos^4(\zeta)] \theta_1 d\zeta$$

When the integral on the left-hand side is performed using Leibniz's integral formula, we obtain

$$\frac{r_0^2}{a_1^2} \frac{d}{dt} \left[ \int_0^{\pi/2} \theta_1 d\zeta \right] = \frac{3\pi\omega_1^2}{16\alpha_1} + \int_0^{\pi/2} [12 \cos^2(\zeta) + (\omega_1^2 - 16) \cos^4(\zeta)] \theta_1 d\zeta \quad (29)$$

(29) is called the energy integral equation for the problem considered here.

Substituting (27) and (28) the above into the energy integral equation (29) we obtain the following ordinary for  $C_1(t)$

$$\frac{r_0^2}{a_1^2} \left( \frac{\pi}{2} - \frac{\tan(\omega_1 \pi/2)}{\omega_1} \right) \frac{dC_1(t)}{dt} = \frac{3\pi\omega_1^2}{16\alpha_1} \left( \frac{1}{\alpha_1} + C_1(t) \right)$$

$$\begin{cases} C_1(0) = 0 \\ A_1(t) = -C_1(t) \sec\left(\omega_1 \frac{\pi}{2}\right) \end{cases}$$

The solution of equation

$$C_1(t) = \exp\left( \left( \frac{\pi}{2} - \frac{\tan(\omega_1 \pi/2)}{\omega_1} \right)^{-1} \frac{3\pi\omega_1^2 a_1^2}{16\alpha_1 r_0^2} t - \ln(\alpha_1) \right) - \frac{1}{\alpha_1}$$

and

$$A_1(t) = \left[ \frac{1}{\alpha_1} - \exp\left( \left( \frac{\pi}{2} - \frac{\tan(\omega_1 \pi/2)}{\omega_1} \right)^{-1} \frac{3\pi\omega_1^2 a_1^2}{16\alpha_1 r_0^2} t - \ln(\alpha_1) \right) \right] \sec\left(\omega_1 \frac{\pi}{2}\right)$$

Finally temperature profile

$$\theta_1(\zeta, t) = \left[ \frac{1}{\alpha_1} - \exp \left( \left( \frac{\pi}{2} - \frac{\tan(\omega_1 \pi/2)}{\omega_1} \right)^{-1} \frac{3\pi\omega_1^2 a_1^2}{16\alpha_1 r_0^2} t - \ln(\alpha_1) \right) \right] \sec \left( \omega_1 \frac{\pi}{2} \right) \cos(\omega_1 \zeta) + \exp \left( \left( \frac{\pi}{2} - \frac{\tan(\omega_1 \pi/2)}{\omega_1} \right)^{-1} \frac{3\pi\omega_1^2 a_1^2}{16\alpha_1 r_0^2} t - \ln(\alpha_1) \right) - \frac{1}{\alpha_1}$$

### Conclusion

The problem (23)-(26) is solved by integral method. All coefficients of temperature profile is found. This method is useful to apply to solving the phase-change problem with moving boundary.

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### ЭЛЛИПСОИДТІК КООРДИНАТТАРДАҒЫ СТЕФАН ЕСЕБІ

**Аннотация.** Бұл мақалада симметриялық электрлік байланыста квазистационарлық Стефан мәселесі берілген. Ерітінді әдісі сызықтық жағдайда стационарлық өрістерге дұрыс болатын контактілерде тең потенциалды және изотермиялық беттердің идентификациясы, сондай-ақ сызықты емес жағдайда да қауіпсіз болуын ұсыныспен алуға болады. Бұл идея цилиндрлік координаттарда эллипсоидтік координаттарға берілген проблема жүйесін өзгерту болып табылады. Стефанның стационарлық мәселесінің аналитикалық шешімі табылды. Осы шешім негізінде эллипсоидтік координаттарда Джоул жылумен жылу проблемасын жуықтап шешу үшін температуралық профиль құрылды.

**Түйіндісөздер:** квази-стационарлық үлгі, Стефан проблемасы, интеграл әдісі

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### ЗАДАЧА СТЕФАНА В ЭЛЛИПСОИДАЛЬНЫХ КООРДИНАТАХ

**Абстрактные.** В настоящей работе представлена квазистационарная задача Стефана в симметричных электрических контактах. Метод решения может быть получен из предположения, что идентичность эквипотенциальных и изотермических поверхностей в контактах, которая правильна для стационарного поля в линейном случае также и для нелинейного случая. Идея состоит в том, чтобы преобразовать систему задач, заданную в цилиндрических координатах, в эллипсоидальные координаты. Получено аналитическое решение стационарной задачи Стефана. На основании этого решения был построен профиль температуры приближенному решению тепловой задачи с Джоулевым нагревом в эллипсоидальных координатах.

**Ключевые слова:** квазистационарная модель, проблема Стефана, интегральный метод.

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