ISSN 2518-1726 (Online),
ISSN 1991-346X (Print)

ҚАЗАҚСТАН РЕСПУБЛИКАСЫ
ҰЛТТЫҚ ҒЫЛЫМ АКАДЕМИЯСЫНЫҢ
Әль-фараби атындағы Қазақ ұлттық университетінің

ХАБАРЛАРЫ

ИЗВЕСТИЯ

НАЦИОНАЛЬНОЙ АКАДЕМИИ НАУК
РЕСПУБЛИКИ КАЗАХСТАН
Казахский национальный университет
имени Аль-фараби

NEWS

OF THE NATIONAL ACADEMY OFSCIENCES
OF THE REPUBLIC OF KAZAKHSTAN
Al-farabi kazakh
national university

SERIES
PHYSICO-MATHEMATICAL

2 (324)

MARCH - APRIL 2019

PUBLISHED SINCE JANUARY 1963

PUBLISHED 6 TIMES A YEAR

ALMATY, NAS RK

Бас редакторы
ф.-м.ғ.д., проф., ҚР ҰҒА академигі **Ғ.М. Мұтанов**

Редакция алқасы:

Жұмаділдаев А.С. проф., академик (Қазақстан)
Кальменов Т.Ш. проф., академик (Қазақстан)
Жантаев Ж.Ш. проф., корр.-мүшесі (Қазақстан)
Өмірбаев У.У. проф. корр.-мүшесі (Қазақстан)
Жүсіпов М.А. проф. (Қазақстан)
Жұмабаев Д.С. проф. (Қазақстан)
Асанова А.Т. проф. (Қазақстан)
Бошкаев К.А. PhD докторы (Қазақстан)
Сұраған Д. корр.-мүшесі (Қазақстан)
Quevedo Hernando проф. (Мексика),
Джунушалиев В.Д. проф. (Қырғыстан)
Вишневский И.Н. проф., академик (Украина)
Ковалев А.М. проф., академик (Украина)
Михалевич А.А. проф., академик (Белорус)
Пашаев А. проф., академик (Әзірбайжан)
Такибаев Н.Ж. проф., академик (Қазақстан), бас ред. орынбасары
Тигиняну И. проф., академик (Молдова)

«ҚР ҰҒА Хабарлары. Физика-математикалық сериясы».

ISSN 2518-1726 (Online), ISSN 1991-346X (Print)

Меншіктенуші: «Қазақстан Республикасының Ұлттық ғылым академиясы» РҚБ (Алматы қ.)
Қазақстан республикасының Мәдениет пен ақпарат министрлігінің Ақпарат және мұрағат комитетінде 01.06.2006 ж. берілген **№5543-Ж** мерзімдік басылым тіркеуіне қойылу туралы куәлік

Мерзімділігі: жылына 6 рет.
Тиражы: 300 дана.

Редакцияның мекенжайы: 050010, Алматы қ., Шевченко көш., 28, 219 бөл., 220, тел.: 272-13-19, 272-13-18, http://physics-mathematics.kz/index.php/en/archive

---

© Қазақстан Республикасының Ұлттық ғылым академиясы, 2019

Типографияның мекенжайы: «Аруна» ЖК, Алматы қ., Муратбаева көш., 75.

Главный редактор
д.ф.-м.н., проф. академик НАН РК **Г.М. Мутанов**

Редакционная коллегия:

Джумадильдаев А.С. проф., академик (Казахстан)
Кальменов Т.Ш. проф., академик (Казахстан)
Жантаев Ж.Ш. проф., чл.-корр. (Казахстан)
Умирбаев У.У. проф. чл.-корр. (Казахстан)
Жусупов М.А. проф. (Казахстан)
Джумабаев Д.С. проф. (Казахстан)
Асанова А.Т. проф. (Казахстан)
Бошкаев К.А. доктор PhD (Казахстан)
Сураган Д. чл.-корр. (Казахстан)
Quevedo Hernando проф. (Мексика),
Джунушалиев В.Д. проф. (Кыргызстан)
Вишневский И.Н. проф., академик (Украина)
Ковалев А.М. проф., академик (Украина)
Михалевич А.А. проф., академик (Беларусь)
Пашаев А. проф., академик (Азербайджан)
Такибаев Н.Ж. проф., академик (Казахстан), зам. гл. ред.
Тигиняну И. проф., академик (Молдова)

«Известия НАН РК. Серия физико-математическая».

ISSN 2518-1726 (Online), ISSN 1991-346X (Print)

Собственник: РОО «Национальная академия наук Республики Казахстан» (г. Алматы)
Свидетельство о постановке на учет периодического печатного издания в Комитете информации и архивов Министерства культуры и информации Республики Казахстан **№5543-Ж**, выданное 01.06.2006 г.

Периодичность: 6 раз в год.
Тираж: 300 экземпляров.

Адрес редакции: 050010, г. Алматы, ул. Шевченко, 28, ком. 219, 220, тел.: 272-13-19, 272-13-18,
http://physics-mathematics.kz/index.php/en/archive

© Национальная академия наук Республики Казахстан, 2019

Адрес типографии: ИП «Аруна», г. Алматы, ул. Муратбаева, 75.

Editor in chief
doctor of physics and mathematics, professor, academician of NAS RK **G.M. Mutanov**

Editorial board:

Dzhumadildayev A.S. prof., academician (Kazakhstan)
Kalmenov T.Sh. prof., academician (Kazakhstan)
Zhantayev Zh.Sh. prof., corr. member. (Kazakhstan)
Umirbayev U.U. prof. corr. member. (Kazakhstan)
Zhusupov M.A. prof. (Kazakhstan)
Dzhumabayev D.S. prof. (Kazakhstan)
Asanova A.T. prof. (Kazakhstan)
Boshkayev K.A. PhD (Kazakhstan)
Suragan D. corr. member. (Kazakhstan)
Quevedo Hernando prof. (Mexico),
Dzhunushaliyev V.D. prof. (Kyrgyzstan)
Vishnevskyi I.N. prof., academician (Ukraine)
Kovalev A.M. prof., academician (Ukraine)
Mikhalevich A.A. prof., academician (Belarus)
Pashayev A. prof., academician (Azerbaijan)
Takibayev N.Zh. prof., academician (Kazakhstan), deputy editor in chief.
Tiginyanu I. prof., academician (Moldova)

News of the National Academy of Sciences of the Republic of Kazakhstan. Physical-mathematical series.

ISSN 2518-1726 (Online), ISSN 1991-346X (Print)

Owner: RPA "National Academy of Sciences of the Republic of Kazakhstan" (Almaty)
The certificate of registration of a periodic printed publication in the Committee of information and archives of the Ministry of culture and information of the Republic of Kazakhstan N 5543-Ж, issued 01.06.2006

Periodicity: 6 times a year
Circulation: 300 copies

Editorial address: 28, Shevchenko str., of. 219, 220, Almaty, 050010, tel. 272-13-19, 272-13-18,
http://physics-mathematics.kz/index.php/en/archive

© National Academy of Sciences of the Republic of Kazakhstan, 2019

Address of printing house: ST "Aruna", 75, Muratbayev str, Almaty

N E W S
OF THE NATIONAL ACADEMY OF SCIENCES OF THE REPUBLIC OF KAZAKHSTAN
PHYSICO-MATHEMATICAL SERIES
ISSN 1991-346X **https://doi.org/10.32014/2019.2518-1726.14**
Volume 2, Number 324 (2019), 69 – 78

UDC 531, 532.133, 621.3.018.72.025.1
IRSTI 29.03.77

K.A.Kabylbekov[1], Kh.K.Abdrakhmanova[2],
A.D.Dasibekov[1], B.Sh. Kedelbaev[1], P.A. Saidakhmetov[1]

[1]M.Auezov South-Kazakhstan State University, Shymkent, Kazakhatan
[2]South-Kazakhstan State Pedagogical University, Shymkent, Kazakhstan
kenkab@mail.ru, khadi_kab@mail.ru, dasibekov.azhibek@mail.ru,
b.sh.kedelbaev@mail.ru, timpf_ukgu@mail.ru

CALCULATION AND VISUALIZATION OF SMALL OSCILLATIONS OF A DOUBLE PLANE PENDULUM

Abstract. The article considers the calculation and visualization of small oscillations of a double plane simple pendulum. It contains the brief derivation of the motion equation and its solutions; the mathematical model of the motion in the form of the system of nonlinear differential equations. Experiments with the double pendulum are performed at various ratios of bodies' masses and initial angles, in particular for three values of $\mu = m_1 / m_2$: μ_1= 0.1, μ_2= 0.2, μ_3= 0.3 when the lengths of the pendulum are $l = l_1 = l_2 = 0.25$m and $g = 9.8$ m/s^2. The angles of pendulum deviation are given in radians. The graphs of small oscillations of the pendulum show that beats occur in the system during which energy cyclically passes from one pendulum to another. When one pendulum almost stops, the other pendulum is at its maximum amplitude. After a while pendulums "exchange their states" and so on. Oscillations with a bigger frequency ω_1 are modulated by lower frequency oscillations with a frequency ω_2.

Key words. Double pendulum, small oscillations, beats, energy exchange, natural frequencies, natural mode.

Introduction. Nowadays all educational institutions of Kazakhstan are provided with computer hardware and software, interactive boards and internet. Almost all teachers have completed language and computer courses for professional development. Hence the educational institutions have all conditions for using computer training programs and models for performing computer laboratory works. In recent years the new computer system for carrying out mathematical calculations MATLAB is being widely used in many universities and engineering institutions throughout the world [1-7]. Unfortunately, the numerical calculations carried out by students are often done by means of the calculator. Modern computers are frequently used only for presentation of the work. Actually students should be able not only to solve these or other engineering problems, but also do them by using modern methods, that is, using personal computers.

Students of the physics specialties 5B060400 and 5B011000 successfully master the discipline "Computer modeling of physical phenomena" which is the logical continuation of the disciplines "Information technologies in teaching physics" and "Use of electronic textbooks in teaching physics". The goal of this discipline is to study and learn the MATLAB program language, acquaintance with its huge opportunities for modeling and visualization of physical processes.

In our early works [8-26] we used the MATLAB system for modeling and visualization of physical processes related with mechanics, molecular physics, electromagnetism and quantum physics. This software has enabled us to solve ordinary differential equations (ODE), visualize equipotential lines of charged conductors system, describe the motion of charged particles in electric, magnetic and gravitational fields and etc.

The present article is devoted to simulation of the double plane pendulum by using the package of MATLAB applied programs.

Double pendulum is, undoubtedly, a real miracle of the nature. The jump of complexity which is observed upon transition from a simple pendulum to a double one is amazing. Oscillations of the simple pendulum are of periodic nature. At small deviations from equilibrium such oscillations are harmonic and obey the sine or cosine law. In case of nonlinear oscillations the period depends on amplitude, but the periodicity of motion is conserved. In other words, the approximation of small oscillations quite perfectly reflects the essential properties of the simple pendulum.

The double pendulum "behaves" itself absolutely differently. At the mode of small oscillations the double pendulum moves in a beat pattern, i.e. with variation of resultant amplitude with time. At the increase of energy the nature of oscillations of the pendulum changes dramatically − oscillations become chaotic. In spite of the fact that the double pendulum can be described by the system of several ordinary differential equations, which is quite deterministic model, the emergence of chaos seems very unusual. This situation reminds the Lorentz system where the deterministic model of three equations also reveals chaotic behavior. Let's perform an experiment with the appendix given below and observe the motion of the double pendulum at various ratios of bodies' masses and initial angles.

At first we will develop the mathematical model of the double pendulum in the form of the system of nonlinear differential equations. Let us begin with the derivation of Lagrange equations.

Lagrange equations. In Lagrangian mechanics the system is described by using the generalized coordinates and the generalized velocities. In the considered problem the angles of deviation of the pendulums α_1, α_2 and angular velocities $\dot{\alpha}_1, \dot{\alpha}_2$ are taken as such variables. Using the specified variables, let us to derive the Lagrangian of the double pendulum and write down differential equations of Lagrange.

The simplified model of the double pendulum is shown in figure 1. Let's consider rods to be weightless. Their lengths are equal to l_1 and l_2. The masses of small bodies (they are presented by balls of finite radius) are m_1 and m_2. It is assumed that there is no friction in suspension points.

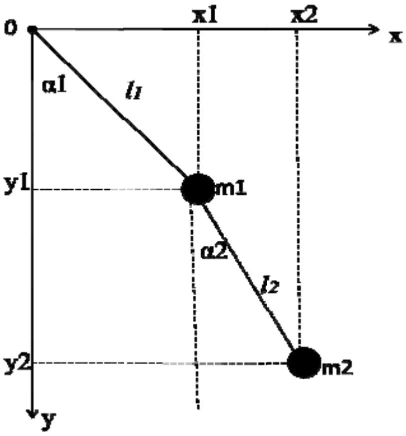

Fig.1 - The simplified model of the double plane pendulum

The masses' coordinates:

$$\begin{aligned} x_1 &= l_1 \sin\alpha_1, \quad x_2 = l_1 \sin\alpha_1 + l_2 \sin\alpha_2, \\ y_1 &= -l_1 \cos\alpha_1, \quad y_2 = -l_1 \cos\alpha_1 - l_2 \cos\alpha_2 \end{aligned} \tag{1}$$

The Lagrange function which is equal to the difference between kinetic and potential energies of the pendulum (respectively T and V) is expressed by the formula:

$$L = T - V = \frac{m_1 \upsilon_1^2}{2} + \frac{m_2 \upsilon_2^2}{2} - V_1 - V_2 =$$
$$= \frac{m_1\left(\dot{x}_1^2 + \dot{y}_1^2\right)}{2} + \frac{m_2\left(\dot{x}_2^2 + \dot{y}_2^2\right)}{2} - m_1 g y_1 - m_2 g y_2 \qquad (2)$$

By taking into account the formula (1) we receive the following expression of the Lagrange function:

$$L = \left(\frac{m_1}{2} + \frac{m_2}{2}\right) l_1^2 \dot{\alpha}_1^2 + \frac{m_2}{2} l_2^2 \dot{\alpha}_2^2 + m_2 l_1 l_2 \dot{\alpha}_1 \dot{\alpha}_2 \cos\left(\alpha_1 - \alpha_2\right) +$$
$$+ m_1 g l_1 \cos\alpha_1 + m_2 g l_1 \cos\alpha_1 + m_2 g l_2 \cos\alpha_2 \qquad (3)$$

Since we consider small oscillations the angles α_1, α_2 are small so the $\cos\left(\alpha_1 - \alpha_2\right)$ can be expanded into Maclaurin series:

$$\cos\left(\alpha_1 - \alpha_2\right) = 1 - \frac{\left(\alpha_1 - \alpha_2\right)^2}{2!} + \frac{\left(\alpha_1 - \alpha_2\right)^4}{4!} - \dots$$

We approximate the function $\cos\left(\alpha_1 - \alpha_2\right)$ by taking the first term of a Maclaurin series since the subsequent terms are negligibly small. With this approximation the equation (3) becomes

$$L = \left(\frac{m_1}{2} + \frac{m_2}{2}\right) l_1^2 \dot{\alpha}_1^2 + \frac{m_2}{2} l_2^2 \dot{\alpha}_2^2 + m_2 l_1 l_2 \dot{\alpha}_1 \dot{\alpha}_2 +$$
$$+ m_1 g l_1 \cos\alpha_1 + m_2 g l_1 \cos\alpha_1 + m_2 g l_2 \cos\alpha_2 \qquad (4)$$

Now we can write down the Lagrange equation:

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\alpha}_i}\right) - \frac{\partial L}{\partial \alpha_i} = 0; \quad i = 1,2 \qquad (5)$$

Do the derivatives:

$$\frac{\partial L}{\partial \dot{\alpha}_1} = \left(m_1 + m_2\right) l_1^2 \dot{\alpha}_1 + m_2 l_1 l_2 \dot{\alpha}_2;$$

$$\frac{\partial L}{\partial \dot{\alpha}_2} = m_2 l_2^2 \dot{\alpha}_2 + m_2 l_1 l_2 \dot{\alpha}_1;$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\alpha}_1}\right) = \left(m_1 + m_2\right) l_1^2 \ddot{\alpha}_1 + m_2 l_1 l_2 \ddot{\alpha}_2;$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\alpha}_2}\right) = m_2 l_2^2 \ddot{\alpha}_2 + m_2 l_1 l_2 \ddot{\alpha}_1;$$

$$\frac{\partial L}{\partial \alpha_1} = -m_1 g l_1 \sin\alpha_1 - m_2 g l_1 \sin\alpha_1 = \left|\sin\alpha \approx \alpha\right| = -m_1 g l_1 \alpha_1 - m_2 g l_1 \alpha_1;$$

$$\frac{\partial L}{\partial \alpha_2} = -m_2 g l_2 \sin\alpha_2 = \left|\sin\alpha \approx \alpha\right| = -m_2 g l_2 \alpha_2;$$

Put it all together to get two Lagrange's differential equations:

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\alpha}_1}\right)-\frac{\partial L}{\partial \alpha_1}=\left(m_1+m_2\right)l_1^2\ddot{\alpha}_1+m_2l_1l_2\ddot{\alpha}_2+\left(m_1+m_2\right)gl_1\alpha_1=0$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\alpha}_2}\right)-\frac{\partial L}{\partial \alpha_2}=m_2l_2^2\ddot{\alpha}_2+m_2l_1l_2\ddot{\alpha}_1+m_2gl_2\alpha_2=0$$

This system of differential equations can be written in a compact matrix form. Firstly, we introduce the following matrixes:

$$\alpha(t)=\begin{pmatrix}\alpha_1(t)\\ \alpha_2(t)\end{pmatrix},\quad M=\begin{pmatrix}\left(m_1+m_2\right)l_1^2 & m_2l_1l_2\\ m_2l_1l_2 & l_2^2m_2\end{pmatrix},$$

$$K=\begin{pmatrix}\left(m_1+m_2\right)gl_1 & 0\\ 0 & m_2gl_2\end{pmatrix},\quad 0=\begin{pmatrix}0\\ 0\end{pmatrix}$$

Then the system of differential equations is expressed as

$$M\ddot{\alpha}+K\alpha=0$$

When there is the oscillation of one body such equation describes free undamped oscillation with a particular frequency. In case of the double pendulum oscillation the solution of this equation (as you will see below) will contain two characteristic frequencies which are called normal or natural modes. Natural modes represent the real part of a complex vector function

$$\alpha(t)=\begin{pmatrix}\alpha_1(t)\\ \alpha_2(t)\end{pmatrix}=\mathrm{Re}\begin{bmatrix}\left(H_1\right)e^{i\omega t}\\ \left(H_2\right)e^{i\omega t}\end{bmatrix}$$

here H_1, H_2 are the eigenvectors, ω is the real value of the frequency. The values of natural frequencies $\omega_{1,2}$ are determined by solving the characteristic equations

$$\det\left(K-\omega^2M\right)=0$$

$$\begin{bmatrix}\left(m_1+m_2\right)gl_1-\omega^2\left(m_1+m_2\right)l_1^2 & -\omega^2m_2l_1l_2\\ -\omega^2m_2l_1l_2 & m_2gl_2-\omega^2m_2l_2^2\end{bmatrix}$$

$$\left(m_1+m_2\right)g^2-\omega^2\left(m_1+m_2\right)\left(l_1+l_2\right)g+\omega^4m_1l_1l_2=0$$

We obtained the biquadratic equation for frequencies ω. Let us calculate the discriminant:

$$D=\left(m_1+m_2\right)^2g^2\left(l_1+l_2\right)^2-4m_1\left(m_1+m_2\right)g^2l_1l_2=$$
$$=g^2\left(m_1+m_2\right)\left[\left(m_1+m_2\right)\left(l_1+l_2\right)^2-4m_1l_1l_2\right];$$

Thus the square of the natural frequencies $\omega_{1,2}$ is equal to

$$\omega_{1,2}^2=\frac{g}{2m_1l_1l_2}\left\{\left(m_1+m_2\right)\left(l_1+l_2\right)\pm\sqrt{\left(m_1+m_2\right)\left[\left(m_1+m_2\right)\left(l_1+l_2\right)^2-4m_1l_1l_2\right]}\right\}$$

This expression is quite lengthy. Therefore we assume that the lengths of both pendulums are the same: $l_1 = l_2 = l$. Then the natural frequencies will be determined by more compact formula

$$\omega_{1,2}^2 = \frac{g}{l}\left[(1+\mu)\pm\sqrt{(1+\mu)\mu}\right], \quad where \quad \mu = \frac{m_2}{m_1}$$

This formula shows that natural frequencies $\omega_{1,2}$ depend only upon the parameter μ (when $g/l = 1$). If masses are identical $m_1 = m_2 = m$, i.e. $\mu = 1$ then the natural frequencies are equal to

$$\omega_{1,2} = \sqrt{\frac{g}{l}\left(2 \pm \sqrt{2}\right)}\cdot$$

Here is the MatLab program for calculation and visualization of the double plane pendulum's oscillation:

```
>> mye=0.1;
>> g=9.8;
>>l=0.25;
>>A=mye+((1+mye)*mye).^0.5;
>>w1=(g/l).^0.5*(1+A).^0.5;
>>w2=(g/l).^0.5*(1-A).^0.5;
>>t=0:0.1:5;
>>alfa1=(-pi/12)*(mye./(1+mye)).^0.5*(cos(w1.*t))+… (pi/12)*(mye./(1+mye)).^0.5*(cos(w2.*t));
>>plot(t,alfa1)
>>grid on
>>hold on
>>alfa2=(pi/12)*cos(w1.*t)+(pi/12)*cos(w2.*t);
>> plot(t,alfa2,'ro-')
>>gtext('alfa2')
>> gtext('alfa1')
>> gtext('mye=0.1')
```

The result is presented in the fig.2

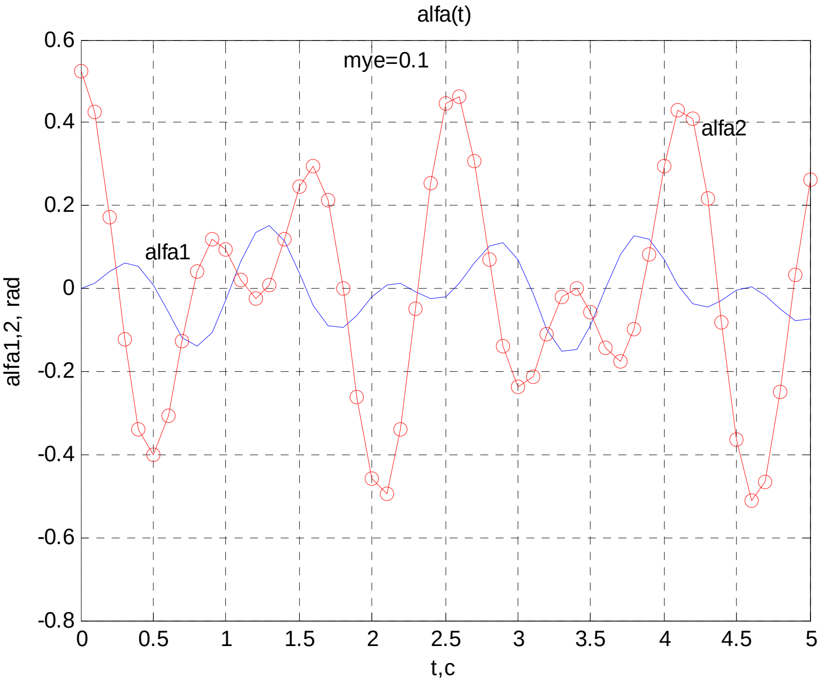

Fig.2 - The graphs of small oscillations of the double pendulum at $\mu_1 = 0.1$

```
>> mye=0.2;
>> g=9.8;
>> l=0.25;
>>   A=mye+((1+mye)*mye).^0.5;
>>  w1=(g/l).^0.5*(1+A).^0.5;
>>  w2=(g/l).^0.5*(1-A).^0.5;
>> t=0:0.1:5;
>> alfa1=(-pi/12)*(mye./(1+mye)).^0.5*(cos(w1.*t))+…
(pi/12)*(mye./(1+mye)).^0.5*(cos(w2.*t));
>>  plot(t,alfa1)
>> grid on
>> hold on
>> alfa2=(pi/12)*cos(w1.*t)+(pi/12)*cos(w2.*t);
>> plot(t,alfa2,'ro-')
>> gtext('alfa2')
>> gtext('alfa1')
>> gtext('mye=0.2')
```

The result is presented in the fig.3

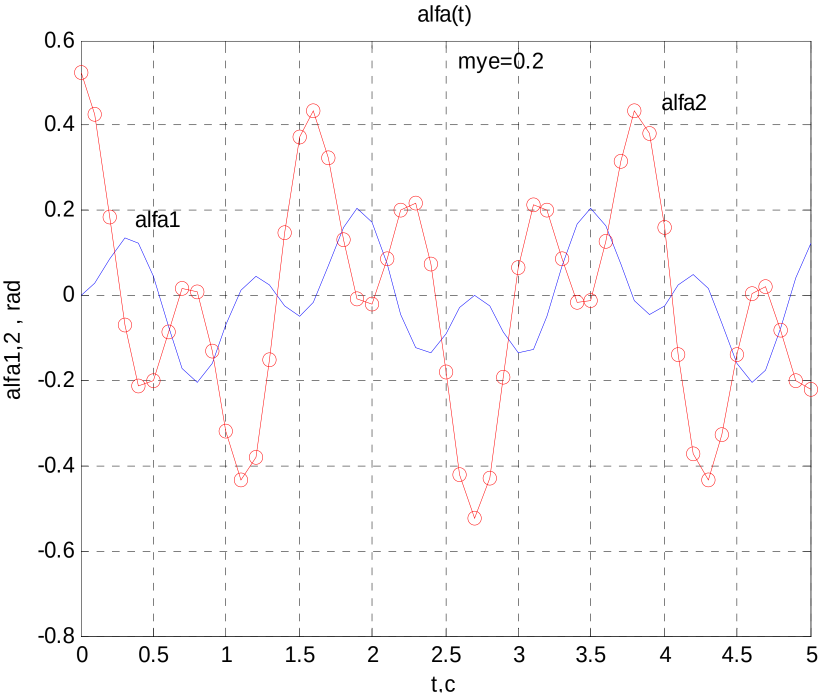

Fig.3 - The graphs of small oscillations of the double pendulum at $\mu_2 = 0.2$

```
>> mye=0.3;
>> g=9.8;
>>l=0.25;
 >> A=mye+((1+mye)*mye).^0.5;
 >> w1=(g/l).^0.5*(1+A).^0.5;
 >> w2=(g/l).^0.5*(1-A).^0.5;
>> t=0:0.1:5;
>>alfa1=(-pi/12)*(mye./(1+mye)).^0.5*(cos(w1.*t))+…
(pi/12)*(mye./(1+mye)).^0.5*(cos(w2.*t));
>> plot(t,alfa1)
>>grid on
>>hold on
```

```
>>alfa2=(pi/12)*cos(w1.*t)+(pi/12)*cos(w2.*t);
>>plot(t,alfa2,'ro-')
>>gtext('alfa2')
>> gtext('alfa1')
>> gtext('mye=0.3')
```

The result is presented in the fig.4

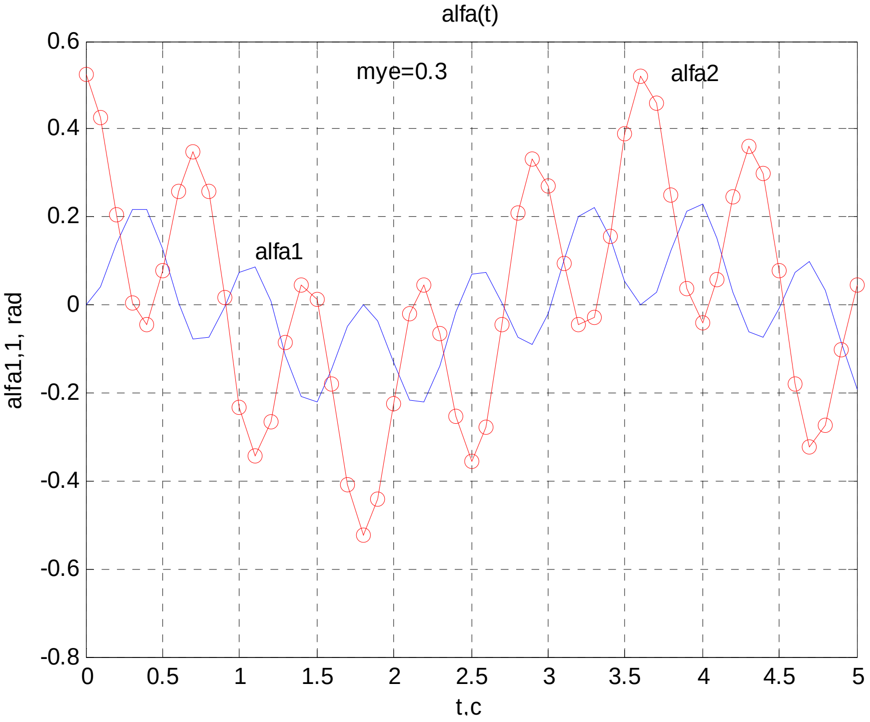

Fig.4 - The graphs of small oscillations of the double pendulum at $\mu_3 = 0.3$

Here angles $\alpha_1(t)$, $\alpha_2(t)$ are expressed in radians, and *t* time is taken in seconds. Figures 2-4 demonstrate the graphs of small oscillations of pendulums for three values of μ: μ_1=0.1, μ_2=0.2, μ_3=0.3 when $l_1 = l_2 = l = 0.25m$, $g = 9.8\ m/s^2$. The angles of displacement of pendulums from equilibrium position are given in radians. It is seen from the graphs that beats occur during double pendulum small oscillations at which energy is cyclically transmitted from one pendulum to the other one. When one pendulum almost stops, the other one is at its maximum amplitude. After a while pendulums "change roles" and so on. Oscillations with a bigger frequency ω_1 are modulated by lower-frequency oscillations with frequency ω_2.

Conclusion. The article considers the calculation and visualization of small oscillations of a double plane simple pendulum. It contains the brief derivation of the motion equation and its solutions; the mathematical model of the motion in the form of the system of nonlinear differential equations. Experiments with the double pendulum are performed at various ratios of bodies' masses and initial angles, in particular for three values of $\mu = m_1 / m_2$: μ_1= 0.1, μ_2= 0.2, μ_3= 0.3 when the lengths of the pendulum are $l = l_1 = l_2 = 0.25$m and $g = 9.8$ m/s^2. The angles of pendulum deviation are given in radians. The graphs of small oscillations of the pendulum show that beats occur in the system during which energy cyclically passes from one pendulum to another. When one pendulum almost stops, the other pendulum is at its maximum amplitude. After a while pendulums "exchange their states" and so on. Oscillations with a bigger frequency ω_1 are modulated by lower frequency oscillations with a frequency ω_2. The results of calculation and visualization of small oscillations of a double plane simple pendulum can be used in the theoretical mechanics.

К.А.Қабылбеков[1], Х.К.Абдрахманова[2],
А.Д.Дасибеков[1], Б.Ш.Кеделбаев[1], П.А.Саидахметов[1]

[1]М.Әуезов атындағы Оңтүстік Қазақстан мемлекеттік университеті, Шымкент, Қазақстан;
[2]Оңтүстік Қазақстан мемлекеттік педагогикалық университеті, Шымкент, Қазақстан

ҚОС ЖАЗЫҚ МАЯТНИКТІҢ АУЫТҚУЫ КІШІ ТЕРБЕЛІСТЕРІН ЕСЕПТЕУ МЕН БЕЙНЕЛЕУ

Аннотация. Қос жазық математикалық маятниктің ауытқуы кіші тербелістерін есептеу мен бейнелеу ұсынылады. Қозғалыс теңдеулерін қорытып шығарылуы мен шешімі келтірілген, сызықтық емес дифференциал теңдеулер бойынша математикалық моделі құрастырылған. Денелердің массаларының әртүрлі қатынастары мен бастапқы ауытқу бұрыштары әртүрлі, атап айтқанда, g = 9.8м/с^2, $l=l_1=l_2=0.25$м шартында, массалар қатынасының үштүрі шамасында $\mu = m_1 / m_2$: $\mu_1=0.1$, $\mu_2=0.2$, $\mu_3=0.3$ жағдайлары үшін қос маятниктің қозғалысын бақылауға арналған эксперименттер жүргізілген. Маятниктердің кіші ребелістерінің графиктері келтірілген. Ауытқу бұрыштары радианда берілген.

Графиктерден жүйеде маятниктердің арасында циклді энергия алмасуы нәтижесінде соққы құбылысы пайда болатыны байқалады. Мятниктің біреуінің ауытқуы тоқтағанда екіншісі максимал амплитудада тербеле бастайды. Әлдебір уақыттан кейін маятниктер рөлі ауысады және осылайша қайталанады. Тербеліс жиілігі ω_1 үлкені жиілігі кіші ω_2 тербеліспен модуляцияланады.

Түйін сөздер. Қос маятник, ауытқуы кіші тербелістер, соққы, энергиямен алмасу, меншікті жиіліктер, нормаль (табиғи) модалар.

К.А.Кабылбеков[1], Х.К.Абдрахманова[2],
А.Д.Дасибеков[1], Б.Ш.Кеделбаев[1], П.А.Саидахметов[1]

[1]Южно-Казахстанский государственный университет им. М.Ауэзова, Шымкент, Казахстан;
[2]Южно-Казахстанский государственный педагогический университет, Шымкент, Казахстан

РАСЧЕТ И ВИЗУАЛИЗАЦИЯ МАЛЫХ КОЛЕБАНИЙ ДВОЙНОГО ПЛОСКОГО МАЯТНИКА

Аннотация. Предлагается расчет и визуализация малых колебаний двойного плоского математического маятника. Приведен краткий вывод уравнения движения и их решения, построена математическая модель в виде системы нелинейных дифференциальных уравнений. Проведены эксперименты по наблюдению за движением двойного маятника при различных отношениях масс тел и начальных углах, в частности для трех значений $\mu = m_1 / m_2$: $\mu_1=0.1$, $\mu_2=0.2$, $\mu_3=0.3$, при условии $l=l_1=l_2=0.25$м, g = 9.8м/с^2. Приведены графики малых колебаний маятников. Углы отклонения маятников приведены в радианах. Из графиков видно, что в системе происходят биения, при которых энергия циклически переходит от одного маятника к другому. Когда один маятник почти останавливается, другой раскачивается с максимальной амплитудой. Через некоторое время маятники "меняются ролями" и так далее. Колебания с большей частотой ω_1 модулируются низкочастотными колебаниями с частотой ω_2.

Ключевые слова. Двойной маятник, малые колебания, биения, обмен энергиями, характерные частоты, нормальные мода.

Information about authors

K.A.Kabylbekov - M. Auezov South Kazakhstan State University (SKSU), Shymkent, Kazakhstan, https://orcid.org/0000-0001-8347-4153, kenkab@mail.ru;

Kh.K.Abdrakhmanova - South Kazakhstan State Pedagogical University (SKSPU), Shymkent, Kazakhstan, https://orcid.org//0000-0002-6110-970X, khadi_kab@mail.ru;

A.D.Dasibekov - M. Auezov South Kazakhstan State University (SKSU), Shymkent, Kazakhstan, htt ps://orcid.org//0000-0002-7148-5506, dasibekov.azhibek@mail.ru;

B.Sh.Kedelbaev- doct. tekhn. sciences, professor, of the department “Microbiologya” of M. Auezov SKSU, https://orcid.org/0000-0001-7158-1488, b.sh.kedelbaev@mail.ru;

P.A. Saidakhmetov .- M. Auezov South Kazakhstan State University (SKSU), Shymkent, Kazakhstan, https://orcid.org/0000-0002-9146-047X, timpf_ukgu@mail.ru

REFERENCES

[1] Porsev S. V. Computer simulation of physical processes in the package MATLAB. M.: Hot Line-Telecom, 2003.592 p.

[2] Kotkin G.A., Cherkassky V.S. Computer modeling of physical processes using MATLAB: Tutorial. / Novosibirsk University.

[3] Lurie M. S., Lurie O. M. Application of the MATLAB program in the study of course of electrical engineering. For students of all specialties and forms of educatio. Krasnoyarsk: SibSTU, 2006. 208 p.

[4] Potemkin V. system of engineering and scientific calculations MATLAB 5.x (in 2 volumes). Moscow: Dialog-MIFI, 1999.

[5] Averyanov G. P., Budkin, Dmitrieva V. V. Design automation. Computer workshop. Part 1. Solving problems of Electrophysics in MATLAB: tutorial. Moscow: MEPhI, 2009. 111 p.

[6] Dyakonov V. p. MATLAB. Complete tutorial. M: DMK Press, 2012. 768 p.

[7] Ryndin E. A., Lysenko I. E. Solving problems of mathematical physics in
Matlab. - Taganrog: TRTU. 2005. - 62 p.

[8]. Kabylbekov K.A., Abdrakhmanova Kh.K., Abekova J., Abdraimov R.T., Ualikhanova B.S. Calculation and visualization of a system-an electron in a deep square potential well, with use of the software package of MATLAB. Proceeding of the III International Scientific and Practical Conference «Topical researches of the World Science» (Iune 28, 2017,Dubai, UAE). №7(23). Vol.I, July 2017, pp 7-13.

[9] Kabylbekov K., Saidullaeva N., Spabekova R., Omashova G, Tagaev N., Bitemirova A., Berdieva M. Model of a blank form for computer laboratory work on research of the speed selector. Journal of Theoretical and Applied Information Technology15th July 2017. Vol.95. No 13 P 2999-3009, c 2005 – ongoing JATIT & LLS. Indexada en Scopus.

[10] Kabylbekov K.A., Omashova G.,Spabekova R., Saidullaeva N., Saidakhmetuv P., Junusbekova S. Management and organization of computer laboratory work in physics education. Espacios. Vol. 38 (Nº 45) Año 2017. Pág. 35. Indexada en Scopus.

[11] Kabylbekov K., Omashova G, Spabekova R, Saidullaeva N, Saidakhmetuv P. Junusbtkova S Management and organization of computer laboratory work in physics education. Espacios. Vol. 38 (Nº 45) Año 2017. Pág. 35. Indexada en Scopus, Google Schollar.

[12] Kabylbekov K.A., Ashirbaev Kh.A., Arysbaeva A.S., Jumagaliyeva A.M. Model of the form of the organization of computer laboratory work in the study of physical phenomena. Modern high technologies, №4, Moscow, 2015, pp 40-43.

[13] Kabylbekov K.A., Madiyarov N.K., Saidakhmetov P.A. Independent design of research tasks of computer laboratory works on thermodynamics. Proceedings of the IX International Scientific and Methodological Conference. Teaching natural sciences (biology, physics, chemistry) mathematics and computer science. Tomsk-2016, pp 93-99.

[14] Kabylbekov K.A, Omashova G. Sh, Spabekova R.S, Saidakhmetov P.A, Serikbaeva G.S, Aktureeva G. Organization of computer laboratory works on study the turn-on and turn-off current of the power supply by using MATLAB software package. Proceedings of the National Academy of Sciences of the Republic of Kazakhstan, Series of Physics and Mathematics, Almaty, No. 3, 2017, pp139-146**.**

[15] Kabylbekov K.A, Omashova G. Sh, Spabekova R.S, Saidakhmetov P.A, Serikbaeva G.S , Aktureeva G. Organization of computer labs for the study of velocity and height distribution of molecules from the Earth's surface by using MATLAB software package. Herald of RK NAS, Almaty, No. 3, 2017, pp 111-119.

[16] Kabylbekov K.A, Ashirbayev H.A, Abdrakhmanova Kh.K, Dzhumagalieva A.I., Kydyrbekova J.B. Organization of laboratory work on study of electric and magnetic fields by using MATLAB software package. Proceedings of the National Academy of Sciences of the Republic of Kazakhstan, Series of Physics and Mathematics, Almaty, No. 3, (313), 2017, pp 206-212.

[17] Kabylbekov K. A., Spabekova R. S., Omashova G.Sh., Abzhapparov A.A., Polatbek A, Serkebayeva S. G. The use of the software package MATLAB for solving problems on bifurcated electrical circuits. Herald of NAS RK, Almaty 2017, No. 4, pp 101-108.

[18] Kabylbekov K. A., Ashirbaev H. A., Abdrakhmanova Kh.K., Dzhumagalieva A. I., Kadyrbekova J. B. Organization of the performance of the laboratory work "Modeling the electric field of a system consisting of a dielectric square and a long charged conductor" by using MATLAB software package. Proceedings of the National Academy of Sciences of the Republic of Kazakhstan, Series of Physics and Mathematics, Almaty 2017, No. 4, pp 252-259.

[19] Kabylbekov K. A., Abdrakhmanova Kh.K., Ermakhanov M.N., Urmashev B.A., Jaткanbayev E.T. Calculation and visualization of a body motion in a gravitational field. N E W S of the National Academy of Sciences of the Republic of Kazakhstan. Series of geology and technical sciences.Volume 4, Number 430 (2018), P 87-98.
https://doi.org/10.32014/2018.2518-170X.28

[20] Kabylbekov K. A., Abdrakhmanova Kh.K., Omashova G.Sh., Lakhanova K.M., Abekova Zh.A. Organization of computer laboratory work "Calculation and visualization of small forced oscillations" N E W S of the National Academy of

Sciences of the Republic of Kazakhstan, Series of geology and technical sciencesю.Volume 4, Number 430 (2018), pp 145-155. https://doi.org/10.32014/2018.2518-170X.28

[21] Kabylbekov K. A., Abdrakhmanova Kh.K., Omashova G.Sh., Kedelbaev B., Abekova Zh.A. Calculation and visualization of electric field of a space –charled sphere. N E W S of the National Academy of Sciences of the Republic of Kazakhstan. Series of geology and technical sciences. Volume 5, Number 431 (2018), pp 201 – 209. https://doi.org/10.32014/2018.2518-170X.28

[22] K. A. Kabylbekov, Kh. K. Abdrakhmanova, P. A. Saidakhmetov, T. S. Sultanbek, B. Sh. Kedelbaev. Calcuiation and visualization of isotopes separation process using MATLAB program. N E W S of the National Academy of Sciences of the Republic of Kazakhstan. Series of geology and technical sciences. Volume 5, Number 431 (2018), 218 – 225. https://doi.org/10.32014/2018.2518-170X.28

[23] Kabylbekov K. A., Abdrakhmanova Kh.K., Saidakhmetov P.A., Musaev J.M., Issayev Ye.B., Ashirbaev Kh.A. Calculatio and vizualization of a body motion under the gravity force and the and the opposing drag. N E W S of the National Academy of Sciences of the Republic of Kazakhstan Series of geology and technical sciences. Volume 6, Number 432 (2018), pp 85– 95. https://doi.org/10.32014/2018.2518-170X.28

[24] Kabylbekov K. A., Abdrakhmanova Kh.K., Saidakhmetov P.A., Kedelbaev B.Sh.,

Abdraimov R. T., Ualikhanova Б.S. Calculation and vizualization of the field caxsial of tht a coaxsial cable carrying steagy current. N E W S of the National Academy of Sciences of the Republic of Kazakhstan Series of geology and technical sciences. Volume 6, Number 432 (2018), pp 55 – 65. https://doi.org/10.32014/2018.2518-170X.28

[25] Kabylbekov K. A., A.D.Dasibekov, Abdrakhmanova Kh.K., Saidakhmetov P.A.,

Issayev E.B., Urmashev B.A. Calculation and vizualization of occillationg systems. N E W S of the National Academy of Sciences of the Republic of Kazakhstan Series of geology and technical sciences. Volume 6, Number 432 (2018), pp 110 – 120. https://doi.org/10.32014/2018.2518-170X.28

[26] Kenzhekhan Kabylbekov, Khadisha Abdrakhmanova, Gaukhar Omashova, Pulat Saidakhmetov, Turlan Sultanbek, Nurzhamal Dausheyeva. A Laboratory on Visualization of Electrostatic and Magnetic Fields. Acta Polytechnica Hungarica Vol. 15, No. 7, 2018, pp 49-70.

Publication Ethics and Publication Malpractice in the journals of the National Academy of Sciences of the Republic of Kazakhstan

For information on Ethics in publishing and Ethical guidelines for journal publication see http://www.elsevier.com/publishingethics and http://www.elsevier.com/journal-authors/ethics.

Submission of an article to the National Academy of Sciences of the Republic of Kazakhstan implies that the described work has not been published previously (except in the form of an abstract or as part of a published lecture or academic thesis or as an electronic preprint, see http://www.elsevier.com/postingpolicy), that it is not under consideration for publication elsewhere, that its publication is approved by all authors and tacitly or explicitly by the responsible authorities where the work was carried out, and that, if accepted, it will not be published elsewhere in the same form, in English or in any other language, including electronically without the written consent of the copyright-holder. In particular, translations into English of papers already published in another language are not accepted.

No other forms of scientific misconduct are allowed, such as plagiarism, falsification, fraudulent data, incorrect interpretation of other works, incorrect citations, etc. The National Academy of Sciences of the Republic of Kazakhstan follows the Code of Conduct of the Committee on Publication Ethics (COPE), and follows the COPE Flowcharts for Resolving Cases of Suspected Misconduct (http://publicationethics.org/files/u2/New_Code.pdf). To verify originality, your article may be checked by the Cross Check originality detection service http://www.elsevier.com/editors/plagdetect.

The authors are obliged to participate in peer review process and be ready to provide corrections, clarifications, retractions and apologies when needed. All authors of a paper should have significantly contributed to the research.

The reviewers should provide objective judgments and should point out relevant published works which are not yet cited. Reviewed articles should be treated confidentially. The reviewers will be chosen in such a way that there is no conflict of interests with respect to the research, the authors and/or the research funders.

The editors have complete responsibility and authority to reject or accept a paper, and they will only accept a paper when reasonably certain. They will preserve anonymity of reviewers and promote publication of corrections, clarifications, retractions and apologies when needed. The acceptance of a paper automatically implies the copyright transfer to the National Academy of Sciences of the Republic of Kazakhstan.

The Editorial Board of the National Academy of Sciences of the Republic of Kazakhstan will monitor and safeguard publishing ethics.

Правила оформления статьи для публикации в журнале смотреть на сайтах:

www:nauka-nanrk.kz

http://physics-mathematics.kz/index.php/en/archive

ISSN 2518-1726 (Online), ISSN 1991-346X (Print)

Редакторы *М. С. Ахметова, Т.А. Апендиев, Д.С. Аленов*
Верстка на компьютере *А.М. Кульгинбаевой*

Подписано в печать 10.04.2019.
Формат 60х881/8. Бумага офсетная. Печать – ризограф.
5,8 п.л. Тираж 300. Заказ 2.

Национальная академия наук РК
050010, Алматы, ул. Шевченко, 28, т. 272-13-18, 272-13-19