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RADIATIVE PROTON CAPTURE ON ^{3}H AT ASTROPHYSICAL ENERGIES AND ITS ROLE IN THE INITIAL STAGE OF STAR FORMATION

Abstract. Calculations of the astrophysical *S*-factor of the proton radiative capture on ^{3}H at energies from 1 keV to 10 MeV in the frame of the Modified Potential Cluster Model (MPCM) with classification of orbital states of nuclear particles according to Young tableaux and isospin were carried out. The explanation of the used phenomenological model MPCM was done. The classification of the p^3H states according to Young tableaux was given. The p^3H interaction potentials and phase shifts of the p^3H elastic scattering were found.

Key words: Nuclear astrophysics; primordial nucleosynthesis; light atomic nuclei; astrophysical energies; radiative capture; thermonuclear processes; potential cluster model; forbidden states, p^3H system.

1. Introduction

The proton capture on ^{3}H reaction is of certain interest from the theoretical and experimental points of view for understanding the dynamics of photonuclear processes with the lightest atomic nuclei at low and ultralow, i.e., astrophysical energies [1]. In addition, it is able, evidently, to attend in the primordial elements nucleosynthesis in the Universe [1-4], leading to the prestellar formation of ^{4}He nuclei. Therefore, experimental studies of this reaction continue, and new data for the total cross section of the proton radiative capture on ^{3}H and the astrophysical *S*-factor in the energy range from 50 keV to 5 MeV [5] and at 12 and 39 keV [6] (center of mass system) has been obtained. These data will use by us further for comparison with the calculation results.

The considering reaction could play a certain role at the prestellar stage of evolution of the Universe [1-3], when at its cooling down to the temperature about ~0.8 MeV, the processes of the primordial nucleosynthesis [7,8] with the formation of stable ^{2}H, ^{3}He, ^{4}He nuclei, and also stable in the first minutes of the Universe formation ^{3}H nucleus became possible.

1. $p + n \rightarrow {}^2H + \gamma$ (Q = 2.225 MeV),
2. ${}^2H + p \rightarrow {}^3He + \gamma$ (Q = 5.494 MeV),
3. ${}^2H + n \rightarrow {}^3H + \gamma$ (Q = 6.257 MeV),
4. ${}^3H + p \rightarrow {}^3He + n$ (Q = -0.764 MeV),
5. ${}^3He + n \rightarrow {}^3H + p$ (Q = 0.764 MeV),
6. ${}^3H + p \rightarrow {}^4He + \gamma$ (Q = 19.814 MeV),
7. ${}^3He + n \rightarrow {}^4He + \gamma$ (Q = 20.578 MeV),
8. ${}^2H + {}^2H \rightarrow {}^3He + n$ (Q = 3.269 MeV),
9. ${}^2H + {}^2H \rightarrow {}^3H + p$ (Q = 4.033 MeV),
10. ${}^2H + {}^3He \rightarrow {}^4He + p$ (Q = 18.353 MeV),
11. ${}^2H + {}^3H \rightarrow {}^4He + n$ (Q = 17.589 MeV),
12. ${}^2H + {}^2H \rightarrow {}^4He + \gamma$ (Q = 23.847 MeV).

This situation could realize at the live time of the Universe about 100 sec, when the number of protons and neutrons was comparable – approximately 0.2 neutrons from the proton number. The epoch of primordial nucleosynthesis is finished approximately at 200 sec [7]; practically all neutrons are already bound into ^{4}He nuclei, the number of which is about 25% relative to ^{1}H nuclei. At that moment the content of ^{2}H and ^{3}He relative to ^{1}H is about at the level of $10^{-4}\div10^{-6}$ [1-8].

Thus, ^{4}He is the last nucleus at the initial stage of nucleosynthesis because heavier nuclei (such as C, O et al.) could be synthesized only in the process of nuclear reactions in stars. The reason is existence of some instability gap for light nuclei (A = 5,6), that, apparently, can't be bridged in the process of primordial nucleosynthesis. In principle, ^{4}He could generate heavier nuclei (A = 7) in $^4\text{He} + {}^3\text{H} \rightarrow {}^7\text{Li} + \gamma$, $^4\text{He} + {}^3\text{He} \rightarrow {}^7\text{Be} + \gamma$ reactions, but the Coulomb barrier for these reactions is about 1 MeV while the kinetic energy of the nuclei at temperature $\sim9\cdot10^8$ K (≈56 кеV) is of the order of 0.1 MeV [4]. The mechanism of prestellar synthesis of ^{4}He lets to explain its abundance in the Universe, confirms a prestellar stage of ^{4}He generation, and corroborates the Big Bang theory.

However, the abundance of ^{3}H at the first 100–200 sec. after the Big Bang can be not much smaller ^{2}H, since the reaction of the neutron capture, in spite of the reduction their number down to 0.2 from the proton number, can go on deuteron at any energies. In addition, the half-life of ^{3}He equals 4500(8) days [9] and do not take a real contribution in the decrease of the number of ^{3}H at the first minutes after the Big Bang.

The quantity of tritium, except process No.3, increases due to reactions No.5 and No.9, but can decrease due to processes No.6 and 11 (at the energy lower than 0.8 MeV reaction No.4 practically does not contribute in decreasing of tritium). Meanwhile, the known *S*-factor of $^2\text{H} + {}^3\text{H} \rightarrow {}^4\text{He} + \text{n}$ reaction (No.11), for example, at energy 10 keV has the value of order 15 MeV b [10], and of the reaction No.6 about $2\cdot10^{-3}$ keV b that can demonstrate its small relative contribution into the formation of ^{4}He. However, the number of deuterons for reaction No.11 approximately by 4-5 orders less than protons taking part into reaction No.6, therefore the real contribution of such reactions in prestellar formation of ^{4}He will be quite comparable. Consequently, the proton capture reaction on ^{3}H can be of certain interest at consideration of astrophysical problems in regard to formations of stable ^{4}He in the process of primordial nucleosynthesis of the Universe.

It should be noted that formation of the modern understanding of the different forming stages of the Universe, about the processes of nucleosynthesis occurring in it, as well as the properties of new stars is still in progress. Therefore, the acquisition of new information on the primordial nucleosynthesis and mechanisms of the Universe formation on the basis of modern concepts of nuclear astrophysics is very urgent and one of the main tasks of a unified cosmological model construction. All this directly applies to the detailed study of p^3H capture reaction in the astrophysical energy region on the basis of modern nuclear model which, as shown below, has already demonstrated its efficiency in the description of the characteristics of almost 30 such reactions.

2. Used model

Earlier in our works [11-16] the possibility of description of astrophysical *S*-factors or total cross-sections of the radioactive capture for three dozens of processes on the basis of two-body potential cluster model (PCM) was shown and also the preliminary results [17] for p^3H-capture at astrophysical energies have been obtained. The calculations of these reactions are carried out on the basis of the modified variant of PCM with forbidden states (FSs) [18] and classification of states according to Young tableaux (MPCM).

The well-defined success of the MPCM in the description of the total cross sections of this type can be explained by the fact that the potentials of the intercluster interaction in the continuous spectrum are constructed on the basis of the known elastic scattering phase shifts or structure of the resonance spectrum levels of the final nucleus, and for the discrete spectrum - on the basis of the main characteristics of the bound states (BSs) of such nuclei: the excited (ES) or the ground (GS) states. These intercluster potentials are based also on the classification of the cluster states according to Young tableaux [19], which enables one to determine the presence and quantity of the FSs in each partial wave. This means finding the number of wave function (WF) nodes in such cluster systems [16].

Furthermore, such potentials permit us to carry out the calculations of some basic characteristics of the considered particles interaction in the elastic scattering processes and reactions. For instance, these can be the astrophysical *S*-factors of the radiative capture reactions [20] or the total cross sections of these reactions [21]. Including radiative capture cross sections at the astrophysical and thermal energy range which has been considered in our previous papers [11-16]. On the basis of such conception we succeeded in the correct description of the total cross sections of the radiative capture processes of almost thirty reactions for light nuclei at thermal and astrophysical energies [11-16].

Therefore, continuing studying the thermonuclear reactions [11-16] on the basis of the MPCM [16] with separation of orbital states according to Young tableaux let us consider description of the astrophysical *S*-factor of the radiative proton capture on ^{3}H at energies of 1 keV–10 MeV and rate of this reaction from $T_9 = 0.05$ to $T_9 = 2$. Preliminary results on *S*-factor of this reaction at astrophysical energies in the frame of the MPCM were given in our previous work [17]. New results for the rate of the proton capture on ^{3}H were obtained here and comparison of our results from [17], published in 1995, and the newest experimental data also published in 1995 year too, given further and do not take into account in our work [17]. For carrying out of the present calculations the potentials of the scattering processes and bound p^3H states were improved and detailed classification orbital states of p^3H system according to Young tableaux and isospin is given. Basic methods and principles of the MPCM used here recently were partially given in [15], and more detailed in book [16].

3. Classification of p^3H states according to Young tableaux

It is known [19] that states with minimal spin in scattering processes in the certain lightest atomic nuclei are mixed with respect to orbital Young tableaux, for example, the singlet state of p^3H system is mixed according to tableaux {4} and {31} [18]. At the same time, this state in the bound form, for example, singlet p^3H channel of ^{4}He is the pure state with Young tableau {4} [18]. In this case we can suppose [19] that BSs and scattering potentials for N^3H (N^3He) states will be different because of the difference of their Young tableaux. Thus, the explicit dependence of the potential parameters at the given moments *L*, *S* and *J* from Young tableaux $\{f\}$ is permitted in this case.

Now, let's give the classification of states, for example, of N^3H (N^3He) systems according to orbital and spin-isospin Young tableaux and demonstrate how to obtain these results. In the general case, the possible orbital Young tableau $\{f\}$ of some nucleus $A(\{f\})$ consisting of two parts $A_1(\{f_1\}) + A_2(\{f_2\})$ is the direct outer product of orbital Young tableaux of these parts $\{f\}_L = \{f_1\}_L \times \{f_2\}_L$ and is determined by the Littlewood theorem [18,19]. Therefore, the possible orbital Young tableaux of the N^3H (N^3He) systems, in which tableau {3} is used for ^{3}H (^{3}He), are the symmetries $\{4\}_L$ and $\{31\}_L$.

Spin-isospin tableaux are the direct inner product of spin and isospin Young tableaux of the nucleus of *A* nucleons $\{f\}_{ST} = \{f\}_S \otimes \{f\}_T$ and for the system with the number of particles not larger than eight are given in [22]. For any of these moments (spin and isospin), the corresponding tableau of the nucleus consisting of *A* nucleons each of which has an angular moment equals 1/2 is constructed as follows: in the cells of the first row the number of nucleons with the moments pointing in one direction, for example, upward, is indicated. In cells of the second row, if it is required, the number of nucleons with the moments directed in the opposite direction, for example, downward, is indicated. The total number of cells in both rows is equal to the number of nucleons in the nucleus. Moments of nucleons in the first row which have a pair in the second row with the oppositely directed moment are compensated and have, therefore, a zero total moment. The sum of moments of nucleons of the first row, which are not compensated by moments of nucleons of the second one, gives the total moment of the whole system [23].

In this case for N^3H (N^3He) cluster systems at the isospin $T = 0$ and the spin $S = 0$, we have tableau $\{22\}_S$ or $\{22\}_T$; and for *S* or $T = 1$, the Young tableau has the form $\{31\}_S$ or $\{31\}_T$. Upon construction of the spin-isospin Young tableau for the triplet spin state of N^3H (N^3He) systems with $T = 1$, we have $\{31\}_S \otimes \{31\}_T = \{4\}_{ST} + \{31\}_{ST} + \{22\}_{ST} + \{211\}_{ST}$, and for the singlet spin state with $T = 0$, we have $\{22\}_S \otimes \{22\}_T = \{4\}_{ST} + \{22\}_{ST} + \{1111\}_{ST}$ [22].

The total Young tableau of the nucleus is determined in a similar way as the direct inner product of the orbital and spin-isospin tableau $\{f\} = \{f\}_L \otimes \{f\}_{ST}$ [19]. The total wave function of the system in the case of antisymmetrization does not identically vanish only if it does not contain the antisymmetric

component $\{1^N\}$, that is realized upon multiplication of conjugated $\{f\}_L$ and $\{f\}_{ST}$. Therefore, the tableaux $\{f\}_L$ conjugated to $\{f\}_{ST}$ are allowed in this channel and all other symmetries are forbidden, since they result to zero total wave function of the system of particles after its antisymmetrization.

Thus, for p^3H system in the triplet channel, independently from the T values, only the orbital wave function with the symmetry $\{31\}_L$ is allowed and the function with $\{4\}_L$ turns out to be forbidden, since the products $\{211\}_{ST} \otimes \{4\}_L$ or $\{31\}_{ST} \otimes \{4\}_L$ do not result in an antisymmetric component of the total wave function. At the same time, in the singlet channel for $T = 0$, we have $\{1111\}_{ST} \otimes \{4\}_L = \{1111\}$ [22], and we obtain the antisymmetric tableau. At this channel with $T = 1$ we have the product $\{211\}_{ST} \otimes \{31\}_L$ that also gives the antysimmetric component $\{1111\}$ for total wave function. Just that very case when one can conclude that the singlet spin state for p^3H and n^3He systems turns out mixed according to Young orbital tableaux each of which relates to different isospin values.

In other words, p^3H system is mixed with respect to isospin, since it has the projection $T_z = 0$, and the following values of the total isospin are possible: $T = 0$ and 1. Hence, in this system both triplet and singlet phase shifts and, therefore, potentials effectively depend on two isospin values. Mixing with respect to isospin leads to mixing according to Young tableaux. As it was shown above, in the singlet spin state two orbital Young tableaux - {31} and {4} - are allowed [17]. Then it was shown in [17,18] that singlet phase shifts of the p^3H scattering mixed with respect to isospin can be represented in the form of the half-sum of pure with respect to isospin singlet phase shifts

$$\delta^{\{T=1\}+\{T=0\}} = 1/2[\delta^{\{T=1\}} + \delta^{\{T=0\}}], \quad (1)$$

this is equivalent to the following expression for the scattering phase shifts in terms of Young tableaux

$$\delta^{\{4\}+\{31\}} = 1/2[\delta^{\{31\}} + \delta^{\{4\}}].$$

Pure phase shifts with Young tableau {31} correspond to $T = 1$, and phase shifts with {4} to isospin $T = 0$. In this approach we assume that pure phase shifts with isospin $T = 1$ in p^3H system can be matched to phase shifts with $T = 1$ for p^3He channel [17,18]. Therefore p^3He system at $T_z = 1$ is pure by isospin with $T = 1$, so the pure by isospin phase shifts of p^3H scattering with $T = 0$ are extracted from expression (1) on the basis of the known pure scattering phase shifts with $T = 1$ for p^3He system [24-29] and for mixed p^3H with isospin with $T = 0$ and 1 [30-32]. Furthermore, the corresponded pure potentials of p^3H interaction are constructed on their basis, for example, for the GS of ^4He in p^3H channel [17,18].

4. Potentials and p^3H scattering phase shifts

For calculations of the photonuclear processes in the considered system the nuclear part of the intercluster potential of p^3He interactions for each partial wave can be expressed in the form

$$V_{JLS\{f\}}(R) = V_0(JLS\{f\})exp[-\alpha(JLS\{f\})R^2] + V_1(JLS\{f\})exp[-\alpha(JLS\{f\})R] \quad (2)$$

with the point-like coulomb term. This potential, as for p^2H system [11], is constructed so that to correctly describe correspondent partial phase shift of p^3He elastic scattering [24-29].

Consequently, the pure with respect to isospin $T = 1$ potentials of p^3He interactions for the elastic scattering processes were obtained, and their parameters are listed in Table 2 [17,18]. The singlet and pure with respect to isospin S phase shift of p^3He elastic scattering, used later for calculation of the singlet p^3H phase shifts with the isospin $T = 0$, is shown by the solid line in Fig. 1a together with the experimental data from works [24-26].

Therefore, there are several different variants of the phase shift analysis of p^3He elastic scattering, for example [24-29], so the parameters of the potential for the singlet $^1P_1$ and the triplet $^3P_1$ waves given in the Table 2 are chosen in order to receive a certain compromise between different results. The singlet $^1P_1$ phase shift of the elastic p^3He scattering with $T = 1$ used in our calculations of the $E1$ transition to the ground state of ^4He in p^3H channel with $T = 0$ is shown in Fig. 1b by the solid line and the experimental data of works [24-29] are given in this figure too.

Table 2 - The singlet potentials of the form (2) for p^3He scattering, pure with respect to isospin T = 1 [17,18]

System	^{2S+1}L	V_0 (MeV)	α (fm^{-2})	V_1 (MeV)	γ (fm^{-1})
p^3He	^1S	-110.0	0.37	+45.0	0.67
	^1P	-15.0	0.1	–	–

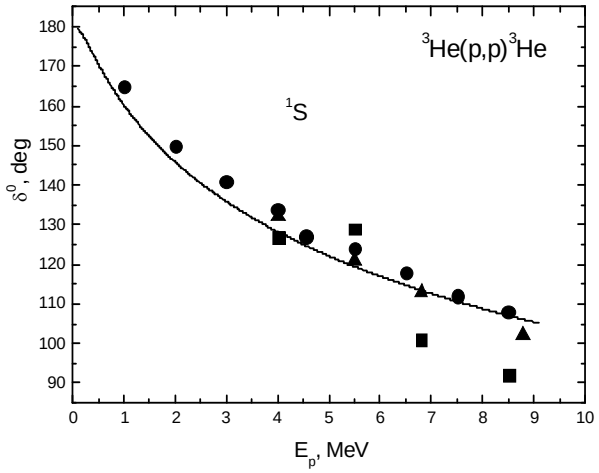

Fig. 1a- Singlet ^1S phase shift of the elastic p^3He scattering. Experimental data: points [24], squares [25], and triangles [26]

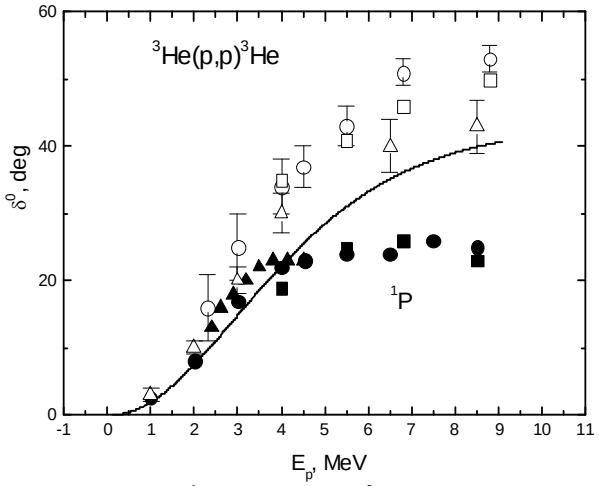

Fig. 1b - Singlet ^1P phase shift of p^3He elastic scattering. Experimental data: points [24], squares [25], triangles [27], circles [28], open squares [26], and open triangles [29]

The singlet and isospin and Young tableaux mixed *S* phase shift of the elastic p^3H scattering, determining from the experimental differential cross sections and used later for obtaining the pure p^3H phase shifts for potential (2) at $V_1 = 0$ with parameters

$$V_0 = -50 \text{ MeV}, \quad \alpha = 0.2 \text{ fm}^{-2}$$

is shown in Fig. 2 by the solid line together with the experimental data of works [30-32].

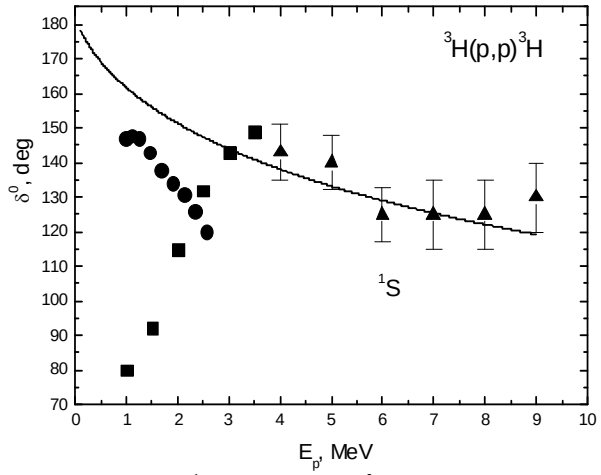

Fig. 2 - Singlet ^1S phase shift of p^3H elastic scattering. Experimental data: points [30], squares [31], and triangles [32]

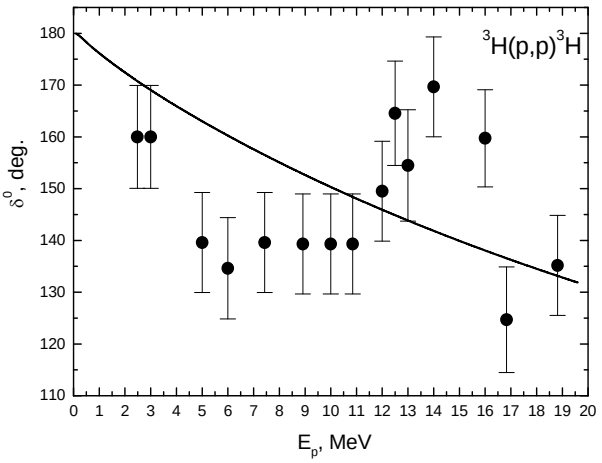

Fig. 3 - Singlet pure according to Young tableau ^1S phase shift of p^3H elastic scattering

Then, using expression (1), for the pure p^3H potential with $T = 0$ in the ^1S wave in [17] at $V_1 = 0$ the following parameters have been found:

$$V_0 = -63.1 \text{ MeV}, \quad \alpha = 0.17 \text{ fm}^{-2}. \qquad (3)$$

Fig. 3 shows the pure with respect to Young tableau singlet ^1S phase of p^3H elastic scattering (dotted line) and the results of calculation of this phase with potential (3) (solid line). Thus-obtained pure (according to Young tableau) interactions can be used for calculation of different characteristics of the bound ground state ^4He in p^3H channel. The degree of agreement of the results obtained in this case with experiment now depends only on the degree of clusterization of this nucleus in the considered channel and here one supposes that this degree is high enough and the spectroscopic factor of such channel will be close to unit.

The interaction potential (3) obtained in [17] on the whole correctly describes the channel binding energy of p^3H system (to several kiloelectronvolts) and the root-mean-square radius of ^4He. With this potential and potential of the ^1P scattering wave from Table 2 with the point-like coulomb term for p^3H system were carried out the calculations of differential [18] and total [17] cross sections of the proton radiative capture on ^3H and the astrophysical *S*-factors at energies down to 10 keV. It should be noted that at that time experimental data for the *S*-factor only was known in the energy region above 700–800 keV [33]. Subsequently, new experimental data were obtained [5] and [6]. Therefore, it is of interest to elucidate whether the potential cluster model with the singlet ^1P potential obtained earlier and refined interaction of the pure ground ^1S state of ^4He is capable of describing this new more accurate data.

Our preliminary results [17,18] have shown that for calculation of the *S*-factor at energies of the order of 1 keV the same conditions as in p^2H system [11] should be satisfied – first of all, the accuracy of finding the binding energy of ^4He in p^3H channel should be increased. New modified programs [16] were used here in order to improve parameters of the potential of the ground state for p^3H system of ^4He (see Table 3), which differ from those presented in [17] by approximately 0.2 MeV. This difference is mainly connected with the application in new calculations of more accurate values of masses of p and ^3H particles [34] and more accurate description of the binding energy of ^4He in p^3H channel.

For this energy based on more accurate values of particle masses [9,34], a value of –19.813810 MeV was obtained; the calculation with the potential considered here gives –19.81381000 MeV. The accuracy of determination of the energy value in this potential using our program based on the finite-difference method (FDM) [16] is 10^{-8} MeV. This accuracy of obtaining the binding energy in p^3H channel of ^4He in the given potential with parameters from Table 3 is confirmed by the calculations on the basis of the variational method (VM), the obtained results are given below.

Table 3 - Pure with respect to isospin of $T = 0$ potentials of form (3) for p^3H interactions in a singlet channel. Here, E_{GS} is the calculated bound ground state energy and E_{exp} is the experimental value of this energy [35]

System	^{2S+1}L	V_0 (MeV)	α (fm^{-2})	E_{GS} (MeV)	E_{exp} (MeV)
p^3H	^1S	-62.906841138	0.17	-19.81381000	-19.813810
	^1P	+8.0	0.03	–	–

The behavior of the "tail" of the numerical wave function (WF) $\chi_L(R)$ for p^3H system bound state at large distances was verified using asymptotic constant (AC) [36,37]

$$\chi_L(r) = \sqrt{2k_0}\ C_W W_{-\eta L+1/2}(2k_0 r), \tag{4}$$

which turned out to be equal to $C_W = 4.52(1)$ at an interval of 5–10 fm for the potential with GS parameters from Table 3. Here $\mu Z_1 Z_2 e^2/(q\hbar^2)$ is the coulomb parameter, where q is the wave number determined by the energy of interacting particles in the initial channel, *L* is the orbital moment, $W_{-\eta L+1/2}(2k_0R)$ is the Whittaker function [37], $k_0 = \sqrt{2\mu\frac{m_0}{\hbar^2}E}$ is the wave number of the GS, *E* is the binding energy in p^3H channel, and the constant $\hbar^2/m_0$ is equal to 41.4686 MeV fm^2, m_0 is the atomic mass unit (amu) [16]. The reduced error of the asymptotic constant is determined by its averaging over mentioned above interval of distances, it is practically stable in the limit of this zone.

The region of the AC C_W stabilization is finding for AC determination starting from the maximum distances that were considered by us, which have order 20–30 fm. This region usually is at distances about

7–12 fm. Inside this region the AC is changed by at least the value of the given relative accuracy, which usually is equal to 10^{-3}. At distances lower than stabilization region for the WF we use its numerical values obtained from the Schrödinger equation solution. At long distances WF $\chi_L(R)$ is calculated from its asymptotic (4) determined by the Whittaker function $W_{-\eta L+1/2}(2k_0r)$ taking into account the AC C_W found in the stabilization region.

Known results on extraction of the asymptotic constant from experimental data give a value of 5.16(13) for p^3H channel [37]. For the asymptotic constant of n^3He system in [37], a value of 5.10(38) has been obtained that is very close to the constant of p^3H channel for our GS potential from Table 3. At the same time, in [36] a value of 4.1 was given for the constant of n^3He system, a value of 4.0 for p^3H. The average value between the results of works [36] and [37] is in a quite agreement with our results for the GS potential from Table 3. Apparently, there is a considerable difference between the data of asymptotic constants. For n^3He system, the constant is in the interval of 4.1–5.5, whereas for p^3H channel it may assume values from 4.0 to 5.3.

We should note here the results of work [38] where the average spectroscopic S_f factor that is equal to 1.59, and the average value of the asymptotic normalizing coefficient A_{NC} (ANC) that is equal to 6.02 $fm^{-1/2}$, were obtained on the basis of calculations with different potentials. Use the known relationship between ANC and dimensional AC C [21,39]:

$$A_{NC}^2 = S \times C^2.$$

Meanwhile, this expression is used for dimensional AC C

$$\chi_L(r) = CW_{-\eta L+1/2}\left(2k_0r\right).$$

This dimensional constant is related with the used by us non-dimensional C_W constant by the following way: $C = \sqrt{2k_0}C_W$. Then, using the given in [38] values of ANC and S_f for dimensional AC C we have found the value of 4.77 $fm^{-1/2}$. In this case $\sqrt{2k_0}$ = 1.30, therefore for dimensionless AC C_W we obtain the value of 3.67, which, nevertheless, slightly less than values obtained here and given in works [36,37]. However, if the spectroscopic factor determines the possibility of certain two-body channel, so scarcely this possibility can be more than unit. Therefore, if one assumes that the value $S_f = 1.0$, so for C_W at $\sqrt{2k_0}$ = 1.30 we obtain 4.63 that agrees acceptably with the given above dimensionless value of 4.52 for the ^1S potential of the ground state from Table 3.

For the charge radius of ^4He, with the potential from Table 3, we have obtained value of 1.78 fm (calculation methods of such radius are described in [16-18]) at the experimental value of ^4He radius 1.671(14) fm [35]. For these calculations we have used the values of the tritium radius of 1.73 fm from [9] and the proton radius of 0.8775 fm from data base [34].

The variational method with the independent variation of parameters with the expansion of the cluster wave function of the p^7Li system in non-orthogonal Gaussian basis [16] is used for an additional control of the accuracy of determination the binding energy in the S potential of the GS from Table 3.

$$\Phi_L(R) = \frac{\chi_L(R)}{R} = R^L \sum_i C_i \exp(-\beta_i r^2), \quad (5)$$

where β – variational parameters and C – expansion coefficients [16]. This method at the basis dimension equals 10 and at independent parameter variation enabled us to obtain the binding energy of -19.81380998 MeV. The asymptotic constant C_W specified by the expression (4) of variational wave function at distances 5–10 fm was survived at level of 4.52(2), and the value of residuals is not more than 10^{-11} [16].

It is known that the variational energy decreases with increasing of the basis dimensionality and gives the upper limit for the true binding energy [40], while the finite-difference energy increases with decreasing of step and increasing of number of steps [16]. Therefore, in the given GS potential in Table 3 for the actual binding energy an average value of –19.81380999(1) MeV can be taken. Thereby, the error of determination of the binding energy in the known GS potential by two methods presented above (FDM

and VM) and based on two different computer programs [16] is ±0.01 eV = ±10 meV or $\pm 10^{-8}$ MeV. Meanwhile it coincides with the initially given accuracy of the FDM, that is equal to 10^{-8} MeV [16].

It can be seen from the given results that the simple two-cluster p^3H model with classification of orbital states according to Young tableaux makes it possible to obtain a quite reasonable values for such characteristics of the bound state of ^{4}He as binding energy, charge radii and asymptotic constants. These results can testify in favor of a relatively high degree of clusterization of this nucleus in p^3H channel. Therefore, such model is completely able to bring us to reasonable results at the calculations of the astrophysical *S*-factors at low and astrophysical energy range.

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РАДИАЦИОННЫЙ ЗАХВАТ ПРОТОНОВ НА ^{3}H ПРИ АСТРОФИЗИЧЕСКИХ ЭНЕРГИЯХ И ЕГО РОЛЬ В НАЧАЛЬНОМ ФОРМИРОВАНИИ ЗВЕЗД

Аннотация. В рамках модифицированной потенциальной кластерной модели (МПКМ) с классификацией орбитальных состояний ядерных частиц по схемам Юнга и изоспину выполнены расчеты астрофизического *S*-фактора реакции радиационного p^3H захвата при энергиях от 1 кэВ до 10 МэВ. Дано объяснение используемой феноменологической модели МПКМ. Приведена классификация p^3H состояний по схемам Юнга. Найдены потенциалы p^3H взаимодействия и фазы p^3H упругого рассеяния.

Ключевые слова: ядерная астрофизика; первичный нюклеосинтез; легкие атомные ядра; астрофизические энергии; радиационный захват; термоядерные процессы; потенциальная кластерная модель; запрещенные состояния, p^3H система.

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АСТРОФИЗИКАЛЫҚ ЭНЕРГИЯ КЕЗІНДЕГІ ^{3}H ПРОТОНДАРДЫҢ РАДИАЦИЯЛЫҚ БАСЫП АЛУ ЖӘНЕ ОНЫҢ ЖҰЛДЫЗДАРЫНЫҢ БАСТАПҚЫ ҚАЛЫПТАСУЫНДАҒЫ РӨЛІ

Аннотация. Түрленген әлеуетті кластерлік модель (ТӘКМ) шеңберінде Юнг сызбалары және изоспин бойынша ядролық бөлшектердің орбиталық жағдайының жіктеуімен 1 кэВ-дан 10 МэВ-ға дейінгі энергия кезінде радиациялық p^3H басып алу реакциясының астрофизикалық S-факторының есептері орындалды. ТӘКМ пайдаланылатын феноменологиялық моделіне түсініктеме берілді. Юнг схемалары бойынша p^3H жағдайының жіктелуі келтірілген. Өзара әрекеттесудің p^3H шамалары және серпінді шашылаудың p^3H фазалары табылды

Түйін сөздер: Ядролық астрофизика; бастапқы нюклеосинтез; жеңіл атом ядролар; астрофизикалық энергия; радиациялық басып алу; термоядролық процесстер; әлеуетті кластерлік модель; тыйым салынған жағдай, p^3H жүйе.

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