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ON LAGRANGE STABILITY AND POISSON STABILITY OF THE DIFFERENTIAL-DYNAMIC SYSTEMS

Abstract. The real difference-dynamic system is considered. The case of unlimited continuity of solutions is investigated for this difference-dynamic system. Unlimited continuity to the right of solutions of the difference-dynamic system is a necessary condition for Lyapunov stability of solutions of this system. Using functions similar to Lyapunov functions, sufficient conditions for the unlimited continuity of solutions of the difference-dynamical system are obtained. The concepts of Lagrange stability and Poisson stability are introduced. Relations between Lyapunov functions and Lagrange stability are investigated. Using the discrete analogue of the second Lyapunov method, the necessary and sufficient conditions for Lagrange stability of the solution of the difference-dynamical system are obtained. A dynamic-difference system is considered for which a discrete operator is constructed, with the help of which the Poisson stability of the system solutions is investigated.

Key Words: difference-dynamic system**,** continuity, Lyapunov functions, Lagrange stability, Poisson stability.

Introduction

Concepts of Lagrange stability and Poisson stability play an important role for studying the qualitative behavior of the trajectories and of the motion of dynamical system given in an arbitrary metric space.

By the Lyapunov functions method necessary and sufficient conditions for the Lagrange stability of systems of ordinary differential equations are obtained in the Ioshizawa work [1]. Poisson stability was investigated from the position of the topological theory of dynamical systems in [2,3]. In these papers a differential operator is constructed in the class of ordinary differential equations. Using this differential operator Poisson stability of solutions of the ordinary differential equations is studied. In [4] the problem of stability of a program manifold with respect to the given vector-function of non-autonomous basic control systems with stationary nonlinearity is investigated.

In this paper the concepts of stability according to Lagrange and Poisson are considered in the class of difference-dynamic systems.

Lagrange stability of the difference dynamic systems

Unlimited continuity of the solutions of the difference-dynamic systems.

Consider the real difference-dynamic systems

$$x_{n+1} = X(n, x_n),\ n \geq 0. \tag{1}$$

There are two possibilities for arbitrary solution $x_n = x(n, n_0, x_0), n_0 \in N^+$:

1) x_n makes sense on the infinite set $N_{n_0} = \{n_0, n_0 + 1, \dots, n, \dots\}$. Then it will continue to the right.;

2) x_n is defined only on some finite interval $\{n_0, n_0 + 1, \dots, m - 1\}$, here $m < \infty$.

Lemma 1. If the solution x_n has a finite time to determine $\{n_0, n_0+1, \ldots, m-1\}, m < \infty$, then

$$\|x_n\| \to \infty \text{ при } n \to m - 0.$$

Proof is obvious, see [5].

Consequence 1. If the solution $x_n = x(n, n_0, x_0)$ is limited on its maximum existence interval $x_n = x(n, n_0, x_0)$, then it remains infinitely continued to the right, i.e. $m = \infty$.

Unlimited continuity to the right of solutions (1) of the difference-dynamic systems is necessary condition for stability according to Lyapunov of solutions of the difference-dynamic systems. Using functions similar to Lyapunov functions, one can obtain sufficient conditions for unbounded continuability of the solutions of the difference-dynamic systems (1) for $n \to +\infty$.

Let us consider the difference-dynamic systems (1). Let x_n be solution with initial condition $x_{n_0} = x^0$. It's clear that

1) This solution can be continued for all $n_0 \le n$. Then the solution x_n can be extended indefinitely.

2) There is $m > n_0$ such that $\|x_n\| \to \infty$ for n → m. Then the solution x_n has a finite definition time.

3) The solution x_n is bounded.

The boundedness of all solutions is stability in a manner. In this case there is Lagrange stability [1,6,7].

In this paper relationships between Lyapunov functions and Lagrange stability (boundedness of solutions) are studied.

Using the discrete analogue of the second Lyapunov method necessary and sufficient conditions for Lagrange stability of the solution of the difference-dynamic system are obtained [1].

Definition 1. Difference-dynamic system (1) is called Lagrange stable if

1) ∃ solutions $x\left(n, n_{0,} x_{n_0}\right)$ for $n \in Z^+$, here $n_0 \in Z^+$;

2) $\left\|x_n\right\|$ is bounded on Z^+.

For example, if the difference-dynamic system (1) has a bounded solution that is asymptotically stable in general. Then difference-dynamic system (1) is Lagrange stable.

Using Lyapunov functions, it is easy to formulate necessary and sufficient conditions for the Lagrange stability of the difference-dynamic systems (1).

Theorem 1. Difference-dynamic systems (1) is Lagrange stable if and only if there exists the function $V\left(n, x_n\right)$ on $Z^+ \times R^k$ such that

1) $V\left(n, x_n\right) \ge W\left(x_n\right)$, here $\lim\limits_{x_n \to \infty} W\left(x_{n_1}\right) = \infty$;

2) the function $V\left(n, x_n\right)$ is not increasing for all solution x_n.

Proof. Sufficiency. Let there is a function $V\left(n, x_n\right)$ with properties 1) and 2) for the difference-dynamic system (1). For all solution $x\left(n; n_0, x_{n_0}\right)$ $\left(n_0 \in Z^+; \left\|x_{n_0}\right\| < \infty\right)$.

By virtue of condition 2), we have $V\left(n, x_n\right) \le V\left(n_0, x_{n_0}\right)$ for $n \ge n_0$.

Using 1) we get

$$W\left(x\left(n; n_0, x_{n_0}\right)\right) \le V\left(n; x\left(n; n_0, x_{n_0}\right)\right) \le V\left(n_0, x_{n_0}\right) \quad \text{for } n \ge n_0 . \tag{2}$$

From the inequality (2) it follows that the solution $x\left(n; n_{0,} x_{n_0}\right)$ is bounded.

Indeed, if this is not the case, then there would be a sequence of moments $n_l \to \infty$ $(l = 1, 2....; n_1 > n_0)$ such that $\lim_{l\to\infty} \|x_{n_l}\| = \infty$. Hence $\lim_{l\to\infty} W x_{n_l} = \infty$.

This would contradict inequality (2), which is impossible.

Necessity. Let arbitrary solution $x_n = x(n; n_0; x_{n_0})$ exists and bounded on Z^+ for the difference-dynamic systems (1).

Set

$$V(n, x_n) = \sup_{v>0} \|x_{n+v}\| = \sup_{v>0} \|x(n+v; n, x_n)\|^2, \tag{3}$$

here $\|x_n\| < \infty, n > n_0 \in Z^+$. From the formula (3) we have $V_n \geq \|x(n+v; n, x_n)\|^2 = \|x_n\|^2 = W(x_n)$.

And, obviously $\lim_{\|x_n\|\to\infty} W(x_n) = \infty$. That is, condition 1) is fulfilled.

Further, considering that due to the uniqueness of the solution, $x_n = x(n; n_2; x_{n_2})$ is a continuation of the solution $x_n = x(n; n_1; x_{n_1})$ for $n_0 < n_1 < n_2$, we have

$$V(n, x_{n_1}) = \sup_{v>0} \|x(n_1 + v; n_1, x_n)\|^2 \geq \sup_{v>0} \left\|x\left(n_2 + v; n_2; x\left(n_2; n_0; x_{n_0}\right)\right)\right\|^2 =$$
$$= V\left(n_2; x\left(n_2; n_0; x_{n_0}\right)\right).$$

Thus, condition 2) is also satisfied.

Poisson stability of the difference dynamic systems

The concept of Poisson stability introduced by A. Poincaré [7] originated in celestial mechanics. This concept represents the broadest concept of periodicity and is characterized by a property called recurrence. Poisson stability is considered as a general concept of the oscillatory regime [6,7].

The problem of Poisson stability was studied from the position of the topological theory of dynamical systems, the main purpose of which, as noted by J. Birkhoff [9], is the classification of movements and the establishment of a connection between them. Such classes include periodicity, almost periodicity in the sense of G. Baire [10], recurrence in the sense of J. Birkhoff [8], almost recurrence in the sense of M.V. Bebutov [11]. The common property of all these classes is the recurrence property, and each individual Poisson stability class is determined by a certain peculiarity of the recurrence character specific for this class. The problem of strong stability and a change in the stability of difference-dynamic system was studied in [12]. Here we consider the difference-dynamic system for which the discrete operator is constructed [2,3], with the help of which the Poisson stability of the solutions of the difference-dynamic system is investigated.

Let us consider the difference-dynamic system

$$x_{n+1} = F(n, x_n). \tag{4}$$

Here $x_n \in R^m$, $n \in Z$, f is vector function, defined and continuous with its partial derivatives $\frac{\partial f}{\partial x_n}$, $k, j = 1, \dots, m$ on $Z \times D$.

The fulfillment of the indicated conditions means that the conditions of the following existence and uniqueness theorem are satisfied for system (4).

Theorem 2. Let

$$(n_0,\ \xi_0) \tag{5}$$

be some point of the set $Z \times D$. Then for all point (5) there is a solution ξ (n) of the difference-dynamic system (4) with the initial condition

$$\xi(n_0) = \xi_0, \tag{6}$$

defined on some interval containing the point n_0. Moreover, if there are two solutions with the same initial condition (6), each of which is defined on its set containing the point n_0, then these solutions coincide in the common area of their definition.

Let

$$x_n = \varphi(n), \tag{7}$$

be solution of the difference-dynamic system (4), defined on $(k, k+1, \dots, k+\tau)$. Let

$$x_n = \psi(n) \tag{8}$$

solution of the same system, but defined on some other interval $(l, l+1, \dots, l+\theta)$. We will say that solution (8) is a continuation of the solution (7), if interval $(l, l+1, \dots, l+\theta)$ contains interval $(k, k+1, \dots, k+\tau)$. Solution (8) coincides with solution (7) on the interval $(k, k+1, \dots, k+\tau)$. In particular, it is considered that solution (8) is a continuation of solution (7) if the interval $(l, l+1, \dots, l+\theta)$ contains interval $(k, k+1, \dots, k+\tau)$. And solution (8) coincides with the solution (7) on $(k, k+1, \dots, k+\tau)$. Solution (8) is a continuation of solution (7) even in the case when the intervals $(l, l+1, \dots, l+\theta)$ and $(k, k+1, \dots, k+\tau)$ coincide, as solutions (7) and (8) completely coincide.

Solution (7) is called noncontinuable if there is no solution different from it, which is its continuation. It is easy to show that each solution can be continued to a non-continuable solution. And there is the only way to do it. Therefore, in the future, only non-continuing solutions will be considered. To emphasize that some solution $\xi(n)$ of the difference-dynamic system (4) is solution with initial condition (6), further we will write this solution in the form

$$\xi(n, n_0, n_0, \xi_0). \tag{9}$$

Then it is easy to see that

1) For each point (5) of the set $Z \times D$, there is a noncontinuable solution of the difference-dynamic system (4) with the initial condition (6).

2) If some noncontinuable solution of the difference-dynamic system (4) coincides with some other noncontinuable solution of this system with at least one value of n, then it is a continuation of this solution.

3) If the two continued solutions of the difference-dynamic system (4) coincide with each other for at least one value of n, then they completely coincide. I.e. they have the same definition interval and they are equal on it.

Let (9) be some non-continuable solution of the difference-dynamic system (4) with the initial condition (7) defined on the interval

$$(k_1(n_0, \xi_0); k_2(n_0, \xi_0)), \tag{10}$$

depending on the initial values (5). Set S is the set of all points (n, n_0, ξ_0) of the space $Z \times Z \times D$ for which solution (9) is defined and satisfies the obvious conditions: point (5) belongs to the set $Z \times D$ and number n to interval (10). Then the following theorem holds, which is well known as the theorem on the continuous dependence of solutions on initial values.

Theorem 3. Set S of all points (n, n_0, ξ_0)) is the open set in the space $Z \times Z \times D$. On the set S the function $\xi(n, n_0, \xi_0)$ is defined. This function is the continued solution (9) of the difference-dynamic system (4) with the initial values (5). At the same time, the function $\xi(n, n_0, \xi_0)$ is continuous across all arguments on S.

We now note that along with the concept of the difference-dynamic system (4) solution, it is often more convenient to use an object very close to it, which we call motion.

Let (9) be a not continued solution defined for all values of $n \in Z$ and let φ be a function given by

$$\varphi(\, n_0, n, \xi_0) = \xi(n_0 + n, n_0, \xi_0). \tag{11}$$

Suppose that under the action of some law mathematically described by the difference-dynamic system (4), the physical system will go into a new state ξ_1 in time $n \in Z^+$. From a mathematical point of view, seems appropriate to determine ξ_1 through the solution of the difference-dynamic system (4) using the formula $\xi_1 = \xi(n_0 + n, n_0, \xi_0)$. From a physical point of view, the record $\xi = \varphi(n_0, n, \xi_0)$ is more appropriate. In this case, obviously, the concept of solution of the difference-dynamic system (4) and the concept of the function φ, corresponding to it, are equivalent.

Definition 2. Let $\varphi(l, n, p)$ be the mapping of the set $Z \times Z^+ \times D$ on the space D. Set $\varphi(l, n, p) = O(l, n)p$ and we will assume that

1) mapping $\varphi(l, n, p)$ is continuous on set of variables l, n, p on $Z \times Z^+ \times D$;
2) there is $O(l, 0)p = p$ for all $(l, p) = Z \times D$;
3) $O(l + s, n)O(l, s) = O(l, n + s)$ takes place for all $(l, n, s) \in Z \times Z^+ \times Z^+$.

Then we say that $\varphi(l, n, p)$ is a motion of a non-autonomous difference-dynamic system [7] if the pair $(l, p) \in Z \times D$ is fixed.

From the above definition it can be seen that the concept of motion is broader than the concept of the solution of difference-dynamic system (4). Thus, in particular, the mapping $\varphi(l, n, p)$ does not have to be differentiable with respect to n, not to mention l, p. At the same time, if in the domain of definition of the solution $x_n = \xi(n, l, p)$ of the difference-dynamic system (4), defined for $n \epsilon N^+$, we take $\varphi(l, n, p) = \xi(l + n, l, p)$, then it is easy to see that the above definitions of motion and solution are very close. Moreover, the motion turns into a solution if $l = 0$. Finally, we call the operator $O(l, n)$ the shift operator along the motion $\varphi(l, n, p)$. It should also be noted here that the form of the shift operator $O(l, n)$ along motions corresponding to the solutions of the difference-dynamic system (1) is determined by the right-hand side of the difference-dynamic system.

Definition 3. A point $p \in D$ is called Poisson positively stable if for each neighborhood A_p and for each positive number T one can specify such number $n \geq T$ that $\varphi(n, p) \in A_p$. Similarly, a point $p \in D$ is called Poisson negatively stable if for each neighborhoods A_p and for every positive number T one can specify such number $n \leq -T$ such that $\varphi(n, p) \in A_p$. And, finally, the Poisson stable point, both positively and negatively, is called Poisson stable [2,5,13]. This is equivalent to the fact that the motion $\varphi(n, p)$ intersects an arbitrary neighborhood A_p for infinitely large n.

Theorem 4. If a point $p \in D$ is positively Poisson stable, then each point of the trajectory, described by the motion $\varphi(n, p)$, is also positively Poisson stable. A similar statement holds for points that are negatively Poisson stable.

Proof. First of all, we note that a point p is positively Poisson stable if and only if there exists such a sequence $n_1, n_2, \dots, n_k, \dots$, $\lim_{k\to\infty} n_k = +\infty$

$$\lim_{k\to\infty} \varphi(n_k, p) = p. \qquad (12)$$

Indeed, the definition of positive stability follows immediately from the equality (12). Conversely, if positive stability holds, there exists a sequence of positive numbers $\varepsilon_1, \varepsilon_2, \dots, \varepsilon_k, \dots$, $\lim_{k\to\infty} \varepsilon_k = 0$ and positive integers $n_k > k$, что $d(p, \varphi(n_k, p)) < \varepsilon_k$, whence follows (12).

Suppose now that p is the arbitrary point of trajectory of the function $\varphi(n, p)$. Then, by virtue of property 1) of definition 2, the equality $\lim\limits_{k\to\infty} \varphi(n + n_k, p) = \varphi(n, p)$ takes place for all values $n \in Z$.

By virtue of Theorem 4, it is easy to see that in the future it makes sense to speak of positive stability and negative stability and Poisson stability not of individual points, but of motions and trajectories of dynamical systems.

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ДИНАМИКАЛЫҚ-АЙЫРЫМДЫҚ ЖҮЙЕЛЕРДІҢ ЛАГРАНЖ ЖӘНЕ ПУАССОН БОЙЫНША ОРНЫҚТЫЛЫҒЫ ТУРАЛЫ

Аннотация. Нақты динамикалық-айырымдық жүйе қарастырылды. Оның шешімдерінің шексіз кеңеюі жағдайы зерттелді. Динамикалық-айырымдық жүйенің шешімдерінің оңға қарай шексіз кеңеюі осы жүйенің шешімінің Ляпунов бойынша орнықтылығы үшін қажетті шарты болып табылады. Ляпуновтың

функциясына ұқсас функцияларды қолдану арқылы біз динамикалық-айырымдық жүйенің шешімдерінің шексіз кеңеюі үшін жеткілікті шарттары алынды. Лагранж бойынша орнықтылығының және Пуассон бойынша орнықтылығының ұғымдары енгізілді. Ляпуновтың функциялары мен Лагранж бойынша орнықтылығы арасындағы қатынастар зерттелді. Ляпуновтың екінші әдісінің дискретті аналогын қолдану арқылы динамикалық-айырымдық жүйенің шешімінің Лагранж бойынша орнықтылығы үшін қажетті және жеткілікті шарттар алынды. Дискретті оператор құрылатын динамикалық-айырымдық жүйе қарастырылды. Құрылған оператордың көмегімен жүйенің шешімдерінің Пуассон бойынша орнықтылығы зерттелді.

Түйін сөздер: динамикалық-айырымдық жүйе, шексіз кеңею, Ляпуновтың функциялары, Лагранж бойынша орнықтылық, Пуассон бойынша орнықтылық.

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ОБ УСТОЙЧИВОСТИ РАЗНОСТНО-ДИНАМИЧЕСКИХ СИСТЕМ ПО ЛАГРАНЖУ И ПО ПУАССОНУ

Аннотация. Рассматривается действительная разностно-динамическая система для которой исследуется случай неограниченной продолжаемости решений. Неограниченная продолжаемость вправо решений разностно-динамической системы является необходимым условием устойчивости в смысле Ляпунова решений этой системы. Используя функции, аналогичные функциям Ляпунова, получены достаточные условия неограниченной продолжаемости решений разностно-динамической системы. Введены понятия устойчивости по Лагранжу и устойчивости по Пуассону. Исследуются соотношения между функциями Ляпунова и устойчивостью по Лагранжу (ограниченностью решений). С помощью дискретного аналога второго метода Ляпунова получены необходимые и достаточные условия устойчивости решения по Лагранжу разностно-динамической системы. Рассматривается разностно-динамическая система для которой строится дискретный оператор, с помощью которой исследуется устойчивость по Пуассону решений системы.

Ключевые слова: разностно-динамическая система, неограниченная продолжаемость, функции Ляпунова, устойчивость по Лагранжу, устойчивость по Пуассону.

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