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# PARTIAL POTENTIALS FOR THE $^8$Li($p$,γ)$^9$Be CAPTURE AT ASTROPHYSICAL ENERGIES

**Abstract.** The total cross sections of the radiative proton capture on $^8$Li at astrophysical energies are considered in the framework of the modified potential cluster model with forbidden states, with the classification of the orbital cluster states according to Young diagrams. The recalculation of the total cross sections for $^9$Be($\gamma$,$p_0$)$^8$Li photodisintegration is used as experimental data. Parameters for Gaussian partial potentials were obtained for description the $^8$Li($p$,γ)$^9$Be capture at astrophysical energies. Simultaneously, in work of Shoda, & Tanaka (1999), different binary channels of disintegration of $^9$Be, namely, $^9$Be(γ,$p$)$^8$Li, $^9$Be(γ,$d$)$^7$Li, $^9$Be(γ,$t$)$^6$Li and also $^9$Be(γ,$^3$He)$^6$He, were experimentally studied. It is evident that the processes of two-body radiative capture connected with them by the detailed balancing principle lead to the synthesis of $^9$Be and require the corresponding estimation contextually in the astrophysical supplements. Meanwhile, it should be noted that the first three reactions have the Coulomb barrier in channels $p^8$Li, $d^7$Li and $t^6$Li lower than in $^3$He$^6$He along with $^4$He$^5$He channels, namely, in the ratio of 3:4. The cross section of $^8$Li($p$,$\gamma$)$^9$Be is hard to obtain directly due to low $^8$Li beam intensity and the small cross section at astrophysical energies. In addition, the difficulty of studying the reaction of $^8$Li($p$,$\gamma$)$^9$Be also lies in the fact that the direct experimental measurement of cross sections is practically impossible due to the very short half-life of $^8$Li – 838 ms. However, as in the case of the neutron capture on $^8$Li, some indirect methods of extracting direct capture cross sections can be used with the help of the radiative capture model and spectroscopic factor.

**Keywords:** Nuclear astrophysics; primordial nucleosynthesis; thermal and astrophysical energies; p$^8$Li cluster system; radiative capture; total cross section.

## 1. Introduction

The study of the formation mechanisms of $^9$Be directly concerns the problem of the overlap of the $A = 8$ mass gap and the synthesis of heavier elements in the early Universe, as well as the $r$-process nucleosynthesis in supernovae (see, for example, [1,2]). In the present time, it is considered that $^9$Be is formed as a result of a two-stage process: the radiative capture of alpha particles $\alpha(\alpha,\gamma)^8$Be, leading to the synthesis of the short-half-life isotope $^8$Be ($t_{1/2} = 6.7\times10^{-17}$s), and radiative neutron capture $^8$Be($n$, γ)$^9$Be [2,3]. In addition, there is the more difficult process of the direct three-particle capture $\alpha\alpha n \rightarrow \gamma\,^9$Be, (see, for example, [4–8]).

Simultaneously, in [8], different binary channels of disintegration of $^9$Be, namely, $^9$Be(γ,$p$)$^8$Li, $^9$Be(γ,$d$)$^7$Li, $^9$Be(γ,$t$)$^6$Li and also $^9$Be(γ,$^3$He)$^6$He, were experimentally studied. It is evident that the processes of two-body radiative capture connected with them by the detailed balancing principle lead to the synthesis of $^9$Be and require the corresponding estimation contextually in the astrophysical supplements. Meanwhile, it should be noted that the first three reactions have the Coulomb barrier in channels $p^8$Li, $d^7$Li and $t^6$Li lower than in $^3$He$^6$He along with $^4$He$^5$He channels, namely, in the ratio of 3:4.

The cross section of $^8$Li($p$,$\gamma$)$^9$Be is hard to obtain directly due to low $^8$Li beam intensity and the small cross section at astrophysical energies. In addition, the difficulty of studying the reaction of $^8$Li($p$,$\gamma$)$^9$Be also lies in the fact that the direct experimental measurement of cross sections is practically impossible due to the very short half-life of $^8$Li – 838 ms [9]. However, as in the case of the neutron capture on $^8$Li

[10], some indirect methods of extracting direct capture cross sections can be used with the help of the radiative capture model and spectroscopic factor [9,11].

The $p^8\mathrm{Li}\rightarrow{}^9\mathrm{Be}\gamma$ reaction presents significant astrophysical interest because it is included in the list of processes of primordial nucleosynthesis of the Universe [12]. However, its experimental investigation in the astrophysical range has so far been insufficient. Intrinsically, there is only one work [13] where the astrophysical range is considered. In [14], it is also added, where measurements were carried out at higher energies. However, these works were published in the 1960s and we do not currently possess more modern experimental studies of the total cross sections of the considered reaction [15]. This is in spite of the fact that studies of spectra $^9$Be in the p$^8$Li channel are still continuing [16]. In addition, the available and considered further theoretical results differ so greatly that it is difficult to draw certain conclusions regarding the rate of this reaction. Furthermore, these calculations do not take into account the existence of resonances in the p$^8$Li system at low energies [16].

In the present study, we consider the reaction of the proton capture on $^8$Li in the frame of the modified potential cluster model (MPCM) [17] and define how the criteria of this model allow us to correctly describe total cross sections and the astrophysical *S*-factor at astrophysical energies. The energy range of 10 keV to 7.0 MeV is considered but only taking into account the structure of resonances up to 2.0 MeV, as discussed previously [16]. The rate of the reaction is calculated at the temperature range of 0.01 to 10 $T_9$. The analysis of the influence of the location and magnitude of resonances to the value and shape of the reaction rate is presented.

## 2. Model and calculation methods

*Further we use well-known formulas for total cross sections and matrix elements of* E*1 transition operators [18] (*$S_i = S_f = S$*)*

$$\sigma_c(NJ,J_f)=\frac{8\pi Ke^2}{\hbar^2 q^3}\frac{\mu\cdot A_J^2(NJ,K)}{(2S_1+1)(2S_2+1)}\frac{J+1}{J[(2J+1)!!]^2}\sum_{L_i,J_i}P_J^2(NJ,J_f,J_i)I_J^2(J_f,J_i), \tag{1}$$

where matrix elements of *EJ* – transitions have a form

$$P_J^2(EJ,J_f,J_i)=\delta_{S_iS_f}[(2J+1)(2L_i+1)(2J_i+1)(2J_f+1)](L_i0J0\,|\,L_f0)^2\begin{Bmatrix}L_i & S & J_i\\ J_f & J & L_f\end{Bmatrix}^2$$

$$A_J(EJ,K)=K^J\mu^J\left(\frac{Z_1}{m_1^J}+(-1)^J\frac{Z_2}{m_2^J}\right),\ I_J(J_f,J_i)=\left\langle\chi_f\middle|r^J\middle|\chi_i\right\rangle \tag{2}$$

Here $S_i$, $S_f$, $L_f$, $L_i$, $J_f$, $J_i$ – are total spins and moments of input (*i*) and output (*f*) channel particles, $m_1$, $m_2$, $Z_1$, $Z_2$ are masses and charges of input channel particles, $I_J$ is the integral over wave functions of the initial $\chi_i$ and final $\chi_f$ states, as relative motion functions of clusters with intercluster distance *r*, μ – reduced mass.

For the spin part of the magnetic process $M1(S)$ at $J = 1$, the following expression is obtained in the model used ($S_i = S_f = S$, $L_i = L_f = L$)

$$P_1^2(M1,J_f,J_i)=\delta_{S_iS_f}\delta_{L_iL_f}[S(S+1)(2S+1)(2J_i+1)(2J_f+1)]\begin{Bmatrix}S & L & J_i\\ J_f & 1 & S\end{Bmatrix}^2,$$

$$A_1(M1,K)=\frac{\hbar K}{m_0c}\sqrt{3}\left[\mu_1\frac{m_2}{m}-\mu_2\frac{m_1}{m}\right],\ I_J(J_f,J_i)=\left\langle\chi_f\middle|r^{J-1}\middle|\chi_i\right\rangle. \tag{3}$$

Here, *m* is the mass of the nucleus, $\mu_1$, $\mu_2$ are magnetic moments of the clusters, and the remaining notation, are given as in the previous expression.

Constant $\hbar^2/m_0$ is equal to 41.4686 MeV fm$^2$, where $m_0$ is the atomic mass unit (amu). The Coulomb potential at zero Coulomb radius $R_{\mathrm{coul}} = 0$ is written in the form $V_{coul}=1.439975\cdot Z_1Z_2/r$, where *r* is the relative distance between particles of the initial channel in fm and *Z* are charges of particles in the elementary charge "*e*" units. Furthermore, the magnetic moment of proton $\mu_p = 2.792847\mu_0$ and $^8$Li nucleus $\mu(^8\mathrm{Li}) = 1.65356\mu_0$ [18], where $\mu_0$ is the nuclear magneton.

## 3. Classification of p$^8$Li states according to Young diagrams

We take Young diagram {431} for $^8$Li – it was shown in [19] that exactly this diagram corresponds to the ground state (GS) of $^8$Li, if to consider it in the $n^7$Li channel. For many other cluster systems, their correspondence to certain Young diagrams were studied by us in [17].

Therefore, for the $p^8$Li system, we have: {431} + {1} = {531} + {441} + {432} + {4311}. The first diagram {531} is forbidden because it cannot be five nucleons in the *s*-shell – it corresponds to orbital momenta $L = 1, 2, 3$, determined by the Elliot rule [20]. The second diagram {441} corresponds to $L = 1, 2, 3, 4$, the third {432} has $L = 1, 2, 3$ and the fourth {4311} corresponds to $L = 0, 2$. For the second diagram {441}, we will consider allowed – it corresponds to the ground state (GS) of $^9$Be in the $p^8$Li channel [21]. Diagrams {432} and {4311} are not considered because without product tables of Young diagrams [22], it is impossible to understand if they are forbidden or allowed.

Thus, it follows from the given classification that for the $p^8$Li system (it is known that $J^\pi,T = 2^+,1$ for $^8$Li [21]) in potentials of $S$ waves there is no forbidden state (FS), in $P$ waves for diagram {441}, there are FS and allowed state (AS), in $D$ and $F$ waves there is FS for the same diagram, which can be considered as bound. The state in the $P_{3/2}$ wave corresponds to the GS of $^9$Be with $J^\pi,T = 3/2^-,3/2$ and lays at the binding energy of $p^8$Li system of -16.8882 MeV [21]. Some $p^8$Li scattering states and bound states (BSs) can be mixed by spin with $S = 3/2$ and $5/2$. However, the same as for the $n^8$Li system [10], here we will consider that the GS of $^9$Be $p^8$Li channel is most probably is the $^4P_{3/2}$ level (in spectroscopic notations $^{2S+1}L_J$).

## 4. Structure of the $p^8$Li resonance states

Let us consider now the state structure of $^9$Be, where we consider further the GS and six low-lying resonance states. The GS of $^9$Be with $J^\pi = 3/2^-$ is at the energy of -16.888 MeV (round off energy down to 1 keV) [16] relatively to the threshold of the $p^8$Li channel and will considered only as the $^4P_{3/2}$ state.

Table 1 - Comparison of data on the $^9$Be spectrum from different works. The threshold of the $p^8$Li channel in $^9$Be equals 16.888 MeV [21]. The results coinciding for all three works are marked by bold

| [16] 2018 year | | | [23] 2016 year | | | [21] 2004 year | | |
|---|---|---|---|---|---|---|---|---|
| $E_r$, MeV | $J^\pi$ | $\Gamma_{c.m.}$, keV | $E_r$, MeV | $J^\pi$ | $\Gamma_{c.m.}$, keV | $E_r$, MeV | $J^\pi$ | $\Gamma_{c.m.}$, keV |
| – | – | – | 0.087(1) | $1/2^-$ | 0.39(1) | 0.0868(8) | $1/2^-$ | 0.389(1) |
| **0.420(7)** | **$5/2^-$** | **210(20)** | **0.410(7)** | **$5/2^-$** | **195** | **0.410(7)** | **$5/2^-$** | **200** |
| **0.610(7)** | **$7/2^+$** | **47(7)** | **0.605(7)** | **$7/2^+$** | **47** | **0.605(7)** | **$7/2^+$** | **47** |
| | | | | | | | | |
| 1.100(30) | $3/2^+$ | 50(22) | 1.132(50) | – | – | 1.132(50) | – | – |
| 1.650(40) | $7/2^-$ | 495(34) | 1.688(30) | $5/2^+$ | 432(50) | 1.692(40) | – | – |
| 1.800(40) | $5/2^-$ | 79(17) | 1.758(40) | $7/2^+$ | 490(81) | 1.762(50) | $5/2^-$ | 300(100) |
| – | – | – | 2.352(50) | $3/2^+$ | 310(80) | 2.312(50) | – | 310(80) |

Here, the $E1$ capture is possible from $^4S_{3/2}$ scattering wave to the $^4P_{3/2}$ GS of $^9$Be, that is, the main contribution is given due to the transition $^4S_{3/2} \to {}^4P_{3/2}$. The spectrum of resonance states of $^9$Be in the cluster $p^8$Li channel is listed in Table 1.

We now discuss this spectrum more closely. Given below, the systematization allows *a priori* estimate of the most significant contribution of resonance states in the capture cross section (the resonance states considered in this work are marked by bold italics).

1. The ***first resonance state*** **(1st RS)** is located at 16.975(8) MeV relative to the GS or 0.0868(8) MeV [21] (0.087(1) MeV [23]) in the center of mass (c.m.) relative to the threshold of the $p^8$Li channel. $J^\pi = 1/2^-$ is given for this level [21,23] that allows us to take $L = 1$ for it, that is, to consider it quartet $^4P_{1/2}$ resonance. The level width of $\Gamma_{c.m.} = 0.39(1)$ keV is given in [21,23]. It is possible to construct absolutely unambiguous $^4P_{1/2}$ potential of the elastic scattering according these data. Ambiguity of its parameters, with the bound FS at the basic variant of state classification by Young diagrams, will be caused only by the error of width of this resonance [17]. The $M1$ transition $^4P_{1/2} \to {}^4P_{3/2}$ to the GS is possible for this state.
2. The ***second resonance state*** **(2nd RS)** is located at 17.298(7) MeV relative to the GS or 0.410(7) MeV in the c.m. relative to the threshold of the $p^8$Li channel [21,23]. $J^\pi = 5/2^-$ is given for this level [21,23] that allows us to take $L = 1$ for it, that is, to consider it quartet $^4P_{5/2}$ resonance. The level width of

$\Gamma_{c.m.}$ = 200 keV is given in [21] and 195 keV in [23]. The energy 420(7) keV at the width 210(20) keV is given in new work [16]. The $M1$ transition $^4P_{5/2}\rightarrow{}^4P_{3/2}$ to the GS is also possible for this state.

3. The *third resonance state* (3rd RS) is located at 17.493(7) relative to the GS or 0.605(7) MeV in the c.m. relative to the threshold of the $p^8$Li channel with the width of 47 keV [21,23]. $J^\pi = 7/2^+$ is given for this level [21,23] that us allows to take $L = 2$ for it, that is, to consider it quartet $^4D_{7/2}$ resonance. The energy 610(7) keV at the width 47(7) keV is given in new work [16]. Since, such resonance state corresponds to the $^4D_{7/2}$ wave, and then only $E2$ transition to the GS of $^9$Be is possible here, which we will not consider due to its small value.

4. The ***fourth resonance state* (4th RS)** at energy 1.100 MeV with the width 50(22) keV and momentum $3/2^+$, which is in data [16], can be considered as refinement of data from [21,23]. Neither its width nor its momentum is not given in them, only the energy of 1.132(50) MeV that approximately coincides with new results from [16] is given. If to take the $^4D_{3/2}$ state for it, the $E1$ transition $^4D_{3/2}\rightarrow{}^4P_{3/2}$ to the GS turn out to be possible. We failed to obtain the resonance in the $^4S_{3/2}$ wave with such characteristics; therefore, we consider this wave nonresonance.

5. The *fifth resonance state* (5th RS) according to [21] is located at the energy of 1.692(40) MeV relative to the threshold, but the momentum and the width are not given for them. The similar state at the energy of 1.688(30) MeV with the momentum $5/2^+$ and the width of 432(50) keV is given in [23]. In new work [16], the energy is equal to 1.650(40) MeV and the width of 495(34) keV that coincide with data from [23], but the value $7/2^-$ is given for momentum. If to assume that the last momentum corresponds to 5th RS, so it can refer to the $^4F_{7/2}$ scattering state, and then only $E2$ transition to the GS is possible, which we will not consider.

6. The *sixth resonance state* (6th RS) according to [21] is located at the excitation energy of 18.65(5) MeV or 1.762(50) MeV in the c.m. relatively to the threshold of the $p^8$Li channel with the width 300(100) keV. $J^\pi = 5/2^-$ is given for this level [21] that allows us to take $L = 1$ for it, that is, to consider it quartet $^4P_{5/2}$ resonance. The $M1$ transition $^4P_{5/2}\rightarrow{}^4P_{3/2}$ to the GS is possible for this resonance. However, the energy of 1.758(40) MeV with the width of 490(81) keV and other momentum $7/2^+$ are given in [23]. At the same time, in new work [16], the energy of 1.800(40) MeV with the width of 79(17) keV and momentum $5/2^-$, coincided with primary data of work [21], are given. If to take for it the last momentum, it can refer to the $^4P_{5/2}$ scattering state and then the $M1$ transition $^4P_{5/2}\rightarrow{}^4P_{3/2}$ to the GS is possible. However, we do not succeed to obtain characteristics of the $P$ wave of continuous spectrum that were noted in Table 1. Therefore, the $F$ scattering wave is compared to it, which gives the $E2$ transition to the GS, but since their contribution is small, we will not consider them.

7. The *seventh resonance state* (7th RS) with the energy of 2.312(50) with the width 310(80) keV and unknown momentum are given in [21]. The energy of 2.352(50) with the width the same width and momentum $3/2^+$ is given in [23]. In new work [16], this and higher states, unfortunately, are not considered. As we have seen above, the results of [23] on two previous levels differ from new data [16]; therefore, the data for this level most probably should be specified, and now we will not consider this resonance.

Slightly higher at excitation energy of 19.420(50) MeV, there is other resonance with the width of 600(100) keV, but it has a presumable momentum $9/2^+$ [23] and cannot lead to $E1$ or $M1$ transitions and we therefore do not consider it. Furthermore, we will base on spectra given in new work [16], including first resonance at 87 keV from [21,23] and limiting by energies not higher the threshold of 2 MeV.

Thus, selected by us, the basic transitions to the GS that are considered here and also $P^2$ coefficients for total cross sections, given in [17], are presented in Table 2. The obtained values of isotopic spin $2T_i$ for resonance states with $J^\pi_i$ are given here.

Table 2 - Characteristics of taken into account transitions at the $^8$Li($p$,$\gamma$)$^9$Be capture

| No. | $[^{2S+1}L_J]_i$ | Resonance energy, MeV | $J^\pi_i$, $2T_i$ | Transition | $[^{2S+1}L_J]_f$, $2T_f = 1$ | $P^2$ |
|---|---|---|---|---|---|---|
| 1. | $^4S_{3/2}$ | – | $3/2^+$ (?) | $E1$ | $^4P_{3/2}$ | 4 |
| 2. | $^4D_{3/2}$ | 1.100 (4th RS) | $3/2^+$, (3?) | $E1$ | $^4P_{3/2}$ | 64/25 |
| 3. | $^4P_{1/2}$ | 0.087 (1st RS) | $1/2^-$, 3 | $M1$ | $^4P_{3/2}$ | 10/3 |
| 4. | $^4P_{5/2}$ | 0.410 (2nd RS) | $5/2^-$, 3 | $M1$ | $^4P_{3/2}$ | 18/5 |

As seen in Table 2, for the **1st RS** (0.087) and the **2nd RS** (0.410), the isospin $T = 3/2$. This means that for these states unambiguously there are no channel mixing $p + {}^8$Li, $d + {}^7$Li, and also $\alpha + {}^5$He, which have the isospin $T = 1/2$. Problems of channels coupling are discussed in [24,25]. Thus, the single-channel approach, which we use here, is quite justified, especially allowing for the fact that capture from **1st RS** (0.087) is highest (see further).

## 5. Criteria of the potential construction

The potentials of resonance waves are constructed in order to correctly describe the location of resonance $E_r$ and its width $\Gamma_{c.m.}$, therefore their parameters are obtained quite unambiguously. GS potentials will be constructed in such a form that allows one to correctly describe the channel binding energy, the charge radius of $^9$Be and its asymptotic constant in the $p^8$Li channel. Since, all known values of the asymptotic normalization coefficient (ANC) and the spectroscopic factor $S_f$, according to which the asymptotic constant (AC) is obtained, have enough large error. The GS potentials also can have few options with different parameters of width. However, at the given values of AC and binding energy, its parameters are constructed absolutely unambiguously.

The radius of $^8$Li equals 2.327±0.0298 fm, which is given in a database [26], was used in further calculations. The radius of 2.518±0.0119 fm for $^9$Be also is known from the database [26]. In addition, for example, the value of 2.299(32) fm for $^8$Li radius was found in [27]. In work [28], for these radiuses, values of 2.30(4) and 2.519(12) fm were obtained, correspondingly. All these data agree well among themselves within the limits of errors. The accurate values of $m(^8\mathrm{Li}) = 8.022487$ amu [29] and $m_p = 1.0072764669$ amu [30] were used for the masses of nucleus and proton. The charge and mass radius of the proton is equal to 0.8775(51) fm [30]. For 1 amu energy equivalent of 931.4941024 MeV was used [30].

The spectroscopic factor $S_f$ of the GS and $A_{NC}$ ANC are connected by the next way [31]:

$$A_{NC}^2 = S_f \times C^2 , \tag{4}$$

where $C$ is the dimensioned asymptotic constant in fm$^{-1/2}$, which connects with dimensionless AC $C_w$ [32] by $C = \sqrt{2k_0}\, C_w$, and the dimensionless constant $C_w$ defined from the relation [32]:

$$\chi_L(r) = \sqrt{2k_0}\, C_w W_{-\eta L+1/2}(2k_0 r), \tag{5}$$

where $\chi_L(r)$ is the numerical BS radial wavefunction, obtained as the solution of the Schrödinger equation normalized to unit size, $W_{-\eta L+1/2}(2k_0 r)$ is the Whittaker function of the bound state, determining the asymptotic behavior of the wavefunction and obtained as the solution of the same equation without nuclear potential, $k_0$ is a wavenumber related to the channel binding energy $E$ where $k_0 = \sqrt{2\mu E/\hbar^2}$ in fm$^{-1}$, $\eta$ is the Coulomb parameter $\eta = \mu Z_1 Z_2 e^2/(\hbar^2 k_0) = 3.44476 \cdot 10^{-2}\, \mu Z_1 Z_2 / k_0$, $Z_1$ and $Z_2$ are the particle charges, $L$ is the orbital momentum of the bound state.

Note that the spectroscopic factor $S_f$ is used by us only for the standard procedure of the obtaining possible $C_w$ range from the obtained in the experiment $A_{NC}$ value [31,32].

## 6. Potentials of the $p^8$Li interaction

As in our previous works [17] for other nuclear systems, we use the potential of the Gaussian form with the point-like Coulomb term with the given orbital momentum $L$ in each partial wave as the $p^8$Li interaction:

$$V(r,L) = -V_L \exp(-\gamma_L r^2). \tag{6}$$

The $^4P_{3/2}$ level we consider as the ground state of $^9$Be in the $p^8$Li and such potential should correctly describe the AC for this channel. In order to extract this constant $C$ in form (4) or $C_w$ (5) from the available experimental data, let us consider information regarding the spectroscopic factors $S_f$ and asymptotic normalization coefficients $A_{NC}$. For example, in [33], except for their own results, the authors add data of previous works. If to separate from these similar results, that is, with closely spaced values of spectroscopic factors that can be presented in the form of table 3.

Table 3 - Data on spectroscopic factors $S_f$ for the GS of $^9$Be in the $p^8$Li channel from works [9,11,33,34]. $\bar{S}_f$ is the average value on data interval

| Reaction from what $S_f$ was determined | $S_f$ for the $p^8\mathrm{Li}_{GS}$ channel | Ref. |
|---|---|---|
| $^8$Li($d$,$n$)$^9$Be | 0.64(21) | [33] |
| $^{12}$C($^9$Be,$^8$Li)$^{14}$N | 0.73(15) | [11] |
| *Average value* | *0.69, that is,* $\sqrt{S_f}$ = 0.83 | |
| *Average value on data interval* | $\bar{S}_f$ *= 0.66(22), 0.43–0.88* $\sqrt{\bar{S}_f}$ *= 0.81(14)* | |
| $^9$Be($^8$Li,$^9$Be)$^8$Li | 1.50(28) | [34] |
| Potential model | 1.50(27) | [9] |
| *Average value on all results* | *1.09,* $\sqrt{S_f}$ *= 1.05* | |
| *Data interval on all results* | $\bar{S}_f$ *= 1.11(68), 0.43-1.78*<br>$\sqrt{\bar{S}_f}$ *= 1.05(28)* | |

Furthermore, the ANC $^4P_{3/2}$ GS in the $p^8$Li channel was given in [35], where $A_{NC}$ = 10.75(12) fm$^{-1/2}$ was obtained. The constant for the $^6P_{3/2}$ state is much less – 0.25(10) fm$^{-1/2}$ [35]. Therefore, we consider here only one spin channel with $S = 3/2$. On the basis of expression (4) and average value on interval $\sqrt{S_f}$ = 0.66(22) from Table 2 according works [11,33] for the AC GS, the value $C$ = 13.71(2.52) fm$^{-1/2}$ was obtained, and because $\sqrt{2k_0} = 1.307$, so dimensionless AC (5) is equal to $C_w$ = 10.49(1.93). However, if to use the average value $\sqrt{S_f}$ on all works from table 3, equals 1.05(28), then for constant $C$, we obtain a wider data interval 11.06(3.06) fm$^{-1/2}$ and $C_w$ = 8.46(2.34). Consequently, the possible interval of $C_w$ values on two data groups from Table 3 is approximately from 6 (from the latest data) to 12.5 (from the previous results).

Furthermore, two options of the GS potentials with FS, which allow us to obtain the dimensionless asymptotic constant $C_w$ in the given above limits, were obtained. The parameters of these potentials $V_L$ and $\gamma_L$, and also main characteristics of the nucleus, obtained with them (binding energy $E_b$, asymptotic constant $C_w$, mass radius $<R>_m$ and charge radius $<R>_{ch}$) are listed in table 4.

Table 4 - GS potential parameters and main characteristics of $^9$Be

| No. | $V_L$, MeV | $\gamma_L$, fm$^{-2}$ | $E_b$, MeV | $C_w$ | $<R>_m$, fm | $<R>_{ch}$, fm |
|---|---|---|---|---|---|---|
| 1 | 212.151135 | 0.17 | -16.88820 | 10.2(1) | 2.40 | 2.46 |
| 2 | 286.178045 | 0.25 | -16.88820 | 6.6(1) | 2.36 | 2.38 |

For example, potential No. 1 has the FS and leads to the binding energy -16.88820 MeV. The definition of the calculation expressions used here for the radii is given, for example, in [17]. The above AC errors are defined by their averaging over the distance interval from 6–8 to 10–12 fm, which is the AC stabilization range. The phase shift of the elastic scattering of this potential for the GS $^4P_{3/2}$ smoothly decreases down to zero and at 5.0 MeV has the value of ~330°. Here, we consider that in the presence of two bound FS and AS, the phase shift according to generalized Levinson theorem starting from 360° [20]. Furthermore, another option of the GS potential that leads to a smaller AC given in table 4.

Table 5 - Options of potential parameters with FS for resonance states of nuclear and some characteristics obtained with them. In the two last columns, the experimental values listed above in Table 1 are shown.

| No. | ${}^{2s+1}L_J$ | $V_J$, MeV | $\gamma_J$, fm$^{-2}$ | $E_r$ (c.m.), keV | $\Gamma_{c.m.}$, keV | $E_r$ (c.m.), keV | $\Gamma_{c.m.}$, keV |
|---|---|---|---|---|---|---|---|
| 1. | ${}^4S_{3/2}$ | -5 | 0.1 | – | – | – | – |
| 2. | ${}^4D_{3/2}$ | 269.242 | 0.2 | 1100 | 56 | 1100(30) | 50(22) |
| 3. | ${}^4P_{1/2}$ | 66.69121 | 0.075 | 87 | ~0.4 | 87(1) | 0.39(1) |
| 4. | ${}^4P_{5/2}$ | 34.0399 | 0.04 | 410 | 203 | 420(7) | 210(20) |

Potential No. 3 from table 5 leads to the ${}^4P_{1/2}$ scattering phase shift, plotted in figure 1 by the black solid curve, which is shown at energies up to 5.0 MeV and has the resonance at 87 keV. Scattering potential No. 4 has the phase shift ${}^4P_{5/2}$ presented in figure 1 by the red dashed curve and at the considered energies has the resonance at 410 keV. Potential No. 2 leads to the ${}^4D_{3/2}$ phase shift shown in figure 1 by the green solid curve. All resonance potentials have the phase shift of 90.0°(1) at the resonance energy and the bound FS.

## 7. Conclusion

It is possible to construct two-body potentials of the $p^8$Li interaction, which allow us to correctly describe the available data on characteristics of the bound state of $^9$Be in the $p^8$Li channel in the frame of the MPCM. Suggested options of the GS potentials of $^9$Be in the $p^8$Li channel allow one to obtain AC within limits of errors available for it and lead to the reasonable description of $^9$Be radii. Such potentials generally allow the available experimental data for total cross sections of the radiative proton capture on $^8$Li at low and ultralow energies. Obtained results for total cross sections and static characteristics of $^9$Be strongly depend of GS potential parameters of this nuclear in the $p^8$Li channel.

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## АСТРОФИЗИКАЛЫҚ ЭНЕРГИЯЛАРДАҒЫ $^8$Li($p,\gamma$)$^9$Be ҚАМТУҒА АРНАЛҒАН ҮЛЕСТІК ПОТЕНЦИАЛДАР

**Аннотация.** Юнга сызбасы бойынша орбиталық кластерлік күйді жіктеумен қатар, тыйым салынған күйдегі модификацияланған потенциалды кластерлі модель аясында, астрофизикалық энергияларда протондарды радиациялық қармауды толық кесу қарастырылған. Эксперименттік мәліметтерді алу үшін $^9$Be($\gamma,p_0$)$^8$Li фотоқирату реакциясын толық кесуді қайта есептеу қолданылады. Астрофизикалық энергияларда $^8$Li($p,\gamma$)$^9$Be радиациялық қармау үшін гаусс потенциалдарының көрсеткіштері анықталды. Сонымен қатар, [Shoda, Tanaka (1999)] $^9$Be: $^9$Be($\gamma,p$)$^8$Li, $^9$Be($\gamma,d$)$^7$Li, $^9$Be($\gamma,t$)$^6$Li, сондай-ақ $^9$Be($\gamma$,$^3$He)$^6$He әртүрлі екілік тарату арналары эксперименталды зерттелді. Олардың егжей-тегжейлі тепе-теңдігімен байланысты екі бөлшекті радиациялық басып алу процестері $^9$Be синтезіне алып келеді және астрофизикалық қосымшалар контекстінде тиісті бағалауды талап етеді. Бұл жағдайда, бірінші үш реакция $p^8$Li, $d^7$Li және $t^6$Li арналарында $^3$He$^6$He-ге қарағанда төмен құлон кедергісінің болуына, яғни 3:4 қатынасында $^4$He$^5$He арналарында, атап айтқанда 3:4 қатынасында назар аудару керек. $^8$Li($p,\gamma$)$^9$Be көлденең қимасы астрофизикалық қызығушылық тудыратын энергия кезіндегі екінші $^{8}$Li шоғырдың және шағын қиманың төмен қарқындылығынан тікелей анықтау қиын. Сонымен қатар, $^{8}\mathrm{Li}(p,\gamma)^{9}\mathrm{Be}$ реакциясын зерттеу мәселесі $^{8}$Li-838 мс ядросының жартылай ыдырауының өте аз кезеңінде қималарды тікелей эксперименталды өлшеу мүмкін емес болып табылады. Алайда, $n^{8}$Li-қармау жағдайында да радиациялық қармау моделін және спектроскопиялық факторды пайдалана отырып, тікелей қармау қимасын алу үшін кейбір жанама әдістер пайдаланылуы мүмкін. $^{9}$Be механизмдердің пайда болуын зерттеу $A = 8$ массалы саңылауды еңсеру және ерте Әлемдегі ауыр элементтердің синтезіне, сонымен қатар supernovae $r$-процесс нюклеосинтез мәселесіне тікелей қатынасы бар. Қазіргі таңда $^{9}$Be екі сатылы үрдіс нәтижесінде пайда болады деген ой қалыптасты: $\alpha(\alpha,\gamma)^{8}\mathrm{Be}$ альфа бөлшектердің радиациалық қармауы қысқа ғұмырлы изотоптың $^{8}$Be ($t_{1/2}$ =6.7×10$^{-17}$ s) синтезіне, одан кейін $^{8}\mathrm{Be}(n,\gamma)^{9}\mathrm{Be}$ нейтронының радиациалық қармауына әкеліп соғады.Аталған мақалада біз төменгі астрофизикалық энергиядағы $p^{8}\mathrm{Li} \to {}^{9}\mathrm{Be}\gamma$ реакциясын қарастырамыз, себебі жарияланғанына 20 жыл уақыт болғанына қарамастан «тәжірибелік зеттерулер жоспары» ретінде рөл атқаратын Terasawa et al. фундаменталды жұмысындағы ауыр элементтердің синтезіне алып келетін мәнді үрдістің тізбегіне қосылған. Leistenschneider et al. (2018) жаңа жұмысында келтірілген 2 МэВ-ке дейінгі резонанс құрылымын ескере отырып 10 кэВ-тен 7.0 МэВ-ге дейінгі энергияның аймағы қарастырылған. Бұл реакцияның жылдамдығы 0.01-ден 10 $T_9$ температура аймағына ғана есептелген. Резонанстар шамалары мен орындарының реакция жылдамдығының формасы мен шамасына әсерінің анализі ұсынылған.

**Түйін сөздер:** Ядролық астрофизика; бастапқы нюклеосинтез; жылулық және астрофизикалық энергиялар; $p^{8}$Li кластерлі жүйесі; радиациялық қармау; толық кесу.

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# ПАРЦИАЛЬНЫЕ ПОТЕНЦИАЛЫ ДЛЯ $^{8}\mathrm{Li}(p,\gamma)^{9}\mathrm{Be}$ ЗАХВАТА ПРИ АСТРОФИЗИЧЕСКИХ ЭНЕРГИЯХ

**Аннотация.** В рамках модифицированной потенциальной кластерной модели с запрещенными состояниями, с классификацией орбитальных кластерных состояний по схемам Юнга рассмотрены полные сечения радиационного захвата протонов на $^{8}$Li при астрофизических энергиях. Перерасчет полных сечений реакции фоторазвала $^{9}\mathrm{Be}(\gamma,p_0)^{8}\mathrm{Li}$ используется для получения экспериментальных данных. Были определены параметры гауссовых потенциалов для радиационного захвата $^{8}\mathrm{Li}(p,\gamma)^{9}\mathrm{Be}$ при астрофизических энергиях. В то же время в [Shoda, Tanaka (1999)] экспериментально исследованы различные бинарные каналы фоторасщепления $^{9}$Be: $^{9}\mathrm{Be}(\gamma,p)^{8}\mathrm{Li}$, $^{9}\mathrm{Be}(\gamma,d)^{7}\mathrm{Li}$, $^{9}\mathrm{Be}(\gamma,t)^{6}\mathrm{Li}$, а также $^{9}\mathrm{Be}(\gamma,{}^{3}\mathrm{He})^{6}\mathrm{He}$. Очевидно, что связанные с ними детальным равновесием процессы двухчастичного радиационного захвата приводят к синтезу $^{9}$Be и требуют соответствующей оценки в контексте астрофизических приложений. При этом, следует обратить внимание на то, что первые три реакции имеют кулоновский барьер в каналах $p^{8}$Li, $d^{7}$Li и $t^{6}$Li ниже, чем в $^{3}$He$^{6}$He равно как и $^{4}$He$^{5}$He каналах, а именно в соотношении 3:4. Поперечное сечение $^{8}\mathrm{Li}(p,\gamma)^{9}\mathrm{Be}$ трудно определить непосредственно из-за низкой интенсивности вторичного $^{8}$Li пучка и малого сечения при энергиях, представляющих астрофизический интерес. Кроме того, проблема изучения реакции $^{8}\mathrm{Li}(p,\gamma)^{9}\mathrm{Be}$ заключается еще и в том, что прямое экспериментальное измерение сечений оказывается практически невозможным из-за очень малого период полураспада ядра $^{8}$Li - 838 мс. Однако, как и в случае $n^{8}$Li-захвата, могут быть использованы некоторые косвенные методы для извлечения сечения прямого захвата с использованием модели радиационного захвата и спектроскопического фактора. Исследование механизмов образования $^{9}$Be имеет прямое отношение к проблеме преодоления массовой щели с $A = 8$ и синтезу более тяжелых элементов в ранней Вселенной, а также в $r$-процесс нюклеосинтеза в суперновых. В настоящее время сложилось мнение, что $^{9}$Be образуется в результате двухступенчатого процесса: радиационный захват альфа-частиц $\alpha(\alpha,\gamma)^{8}\mathrm{Be}$ приводит к синтезу короткоживущего изотопа $^{8}$Be ($t_{1/2}$ =6.7×10$^{-17}$ s), и далее радиационный захват нейтрона $^{8}\mathrm{Be}(n,\gamma)^{9}\mathrm{Be}$. В данной статье мы рассматриваем реакцию $p^{8}\mathrm{Li}\to{}^{9}\mathrm{Be}\gamma$ при низких астрофизических энергиях в связи с тем, что она включена в цепочку значимых процессов, которые приводят к синтезу более тяжелых элементов в фундаментальной значимой работе Terasawa et al., которая с момента ее опубликования следующие почти 20 лет играет роль некоторого «плана практических исследований». Рассмотрена область энергий от 10 кэВ до 7.0 МэВ, но с учетом структуры резонансов только до 2 МэВ, которая была приведена в новой работе Leistenschneider et al. (2018). Скорость этой реакции рассчитана в области температур от 0.01 до 10 $T_9$. Представлен анализ влияния положения и величины резонансов на величину и форму скорости реакции.

**Ключевые слова:** Ядерная астрофизика; первичный нюклеосинтез; тепловые и астрофизические энергии; кластерная система $p^8$Li; радиационный захват; полное сечение.

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