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OPTIMAL CONTROL OF POWER SYSTEMS

Abstract. This article discusses the study of problems of optimal control for electric power systems. The numerical solution of optimal control problems for complex electric power systems using an iterative algorithm is shown. Also considered are issues of solving the optimal control of a nonlinear system of ordinary differential equations in two different cases. The proposed solution methods follow the principle of continuation of extremal problems based on sufficient conditions for optimality of V. F. Krotov. A special case of optimal control problems is considered. Numerical experiments showed sufficient efficiency of the implemented algorithms. The problem of optimal motion control of a two-system electric power system is graphically illustrated in the proposed numerical example.

Keywords: optimal control, electric power systems, an iterative algorithm.

1 Introduction

The industrial growth of any country largely depends on the reliability of a large interconnected power system. The electric power system is an important form of modern energy source, since it is used in almost all spheres of human activity for socio-economic development. In an interconnected power system, the purpose of an electric power system's engine is to generate electrical energy in sufficient quantities at the most appropriate place of generation, transfer it in large quantities to load centers, and then distribute it to individual consumers in the proper order.

Mathematical model of modern electric power complex, consisting of turbo generators and complex multiply connected power units, is a system of nonlinear ordinary differential expressions. It is known [1-3] that this model serves as the basis of a broad and relevant class of control problem.

It should be noted that mathematical simulation of various processes and systems, including electric power system, are closely related with problem of making the best decisions. The optimization problems, as well as the creation of methods of building control on the principle of feedback for such systems, still attract attention of many researchers.

Optimal control theory is based on the maximum principle of L.S. Pontryagin and the method of dynamic programming of R. Bellman. It is known that the maximum principle reduces extreme challenge to the decision of a special system of ordinary differential equations, and dynamic programming methods to the solution of partial differential equations.

In many cases, the exact solution of these tasks is quite difficult. Why we developed numerical methods for solving extreme problems [5], based on the extension principle [6-11], which differ in the considerable variety of approaches and results.

These methods found a wide and effective application in solving some optimal control problems of large dimension and of various complexities [12-23]. Note that our works [24-26] were also devoted to the solution of optimal control problems. A study of global asymptotic stability was carried out in [27].

There are two widely spread areas in engineering practice among different research directions in optimal control theory based on the method of state space. One of them combines the methods of optimal control, which involve optimizing the system by minimizing the functional that characterizes, as a rule, the quality of regulation [21]. The second area contains the methods of modal control, i.e. methods of forming

feedback circuits, giving a closed-loop automatic control system (ACS), pre-selected root distribution of the characteristic equation [28].

The need to reconfigure the values of fudge factors of industrial controllers due to several factors associated with changes in the characteristics of complex energy facilities. Factors occur due to load changes, properties of energy resources, work of parallel regulation channels associated with controller via the object, equipment deterioration, impact of uncontrolled external disturbances, etc. For example, the load change of thermal power unit causes the change of position of regulating units. Tilt of operating characteristics of the regulating units of different types (slide, dampers, dampers, valves, etc.) may change in 2-3 times in different positions. The gain ratio of the object changes in 2-3 times accordingly that results in deteriorating the quality of transition processes. These degradations essentially influence the technical-and-economic performance of the equipment by decreasing its efficiency.

Testing parameters similarly change in other technical processes of complex objects of power-supply branch. In order to increase the effectiveness of the disturbance control and suppression processes, which are caused by the change of equipment operation load and other performance factors, it needs to use optimal digital control systems.

Thus, the study of modern principles of optimal control systems of complex objects is an actual scientific-technical problem.

In this paper, solving of optimal control problems for power system uses the principle of expansion extreme problems based on sufficient optimality conditions.

2 The optimal control formulation

It is required to minimize the functional

$$J(u) = 0.5 \sum_{i=1}^l \int_0^T (k_i y_i^2 + r_i u_i^2) dt + \Lambda(x(T), y(T)), \quad (1)$$

under the condition:

$$\begin{aligned} \frac{dx_i}{dt} &= y_i, & \frac{dy_i}{dt} &= -\lambda_i y_i + f_i(x) + b_i u_i, \\ x_i(0) &= x_{i0}, y_i(0) = y_{i0}, i = \overline{1, l}, t \in (0, T), \\ x(t), y(t) &: (0, T) \rightarrow R^l, \end{aligned} \quad (2)$$

where $\{x_i, y_i\}_{i=1}^l$ – is system condition $\{u_i\}_{i=1}^l$ – control; $\{f_i(x)\}_{i=1}^l, \Lambda(x, y)$ – given continuously differentiable functions and functions

$f_i(x)$ satisfy the integrability conditions:

$$\frac{\partial f_i(x)}{\partial x_k} = \frac{\partial f_k(x)}{\partial x_i}, \forall i \neq k; \quad (3)$$

We consider point in time T and initial states $\{x_{i0}, y_{i0}\}$ ordered; r_i, λ_i, k_i, b_i – positive constants; terminal values $x(T), y(T)$ are unknown earlier.

We should note that if we appropriately set the $f_i(x), i = 1, \dots, l$, – function, non-linear problem of Cauchy (1)-(2) images the electric power system, for which the problem of synthesis is an important practical task of optimal control.

Special case of the control problem (1)-(2).

Further, in the optimal control problem (1)-(2) we assume that there are following data for the control problem:

$$k_i = 2\lambda_i + \frac{b_i^2}{r_i}, \quad i \in \overline{1, l}.$$

In this case, we can solve the problem (1)-(2), following the Bellman-Krotov formalism [9,10]. At first, we show the correctness of the next Lemma.

Lemma 1. In order that the control $u_i^0(y_i) = -\frac{b_i}{r_i} y_i, i = \overline{1, l}$ and the relevant solution of system (2)-(3) $\{x(t), y(t)\}$ could be optimal, it is necessary and sufficient that

$$\varphi(x(T), y(T)) = -\Lambda(x(T), y(T)), k_i = 2\lambda_i + \frac{b_i^2}{r_i}, i \in \overline{1, l}. \quad (4^0)$$

$$\varphi(x, y) = 0.5 \sum_{i=1}^l y_i^2 - \int_{x_j=0, j>i}^l \int_{i=1}^{x_i} f_i(x_1, \dots, x_{i-1}, \xi_i, x_{i+1}, \dots, x_l) d\xi_i \quad (4)$$

$\varphi(x, y)$ – the Bellman-Krotov function, where

$$J(u^0) = \min_u J(u) = -\varphi(x(t_0), y(t_0)).$$

Implementation of iterative algorithm for the problem (1)-(2).

Let us describe the procedure of improving a given s-order approximation

$$\vartheta_s(t) = \{x_{1,s}(t), \dots, x_{l,s}(t), y_{1,s}(t), \dots, y_{l,s}(t), \dots, u_{1,s}(t), \dots, u_{l,s}(t)\}.$$

Step 1. Let us find a solution the next dual problem

$$\begin{cases} \frac{d\psi'_{i,s}(t)}{dt} = -\frac{\partial H(x_s(t), y_s(t), \nabla_{x,y}\varphi(x_s(t), y_s(t), t))}{\partial x_i}, \\ \frac{d\psi'_{l+i,s}(t)}{dt} = -\frac{\partial H(x_s(t), y_s(t), \nabla_{x,y}\varphi(x_s(t), y_s(t), t))}{\partial y_i}, i = 1, \dots, l, \\ \psi_{i,s}(T) = -\frac{\partial \Lambda(x(T), y(T))}{\partial x_i(T)}, \psi_{l+i,s}(T) = -\frac{\partial \Lambda(x(T), y(T))}{\partial y_i(T)}, i = 1, \dots, l, \end{cases}$$

where

$$\begin{aligned} H(x_s(t), y_s(t), \nabla_{x,y}\varphi(x_s(t), y_s(t), t)) &= \max_u H(x_s(t), y_s(t), \nabla_{x,y}\varphi(x_s(t), y_s(t), t), u, t), \\ &= -0.5[k_i y_i^2 + r_i u_i^2] + \sum_{i=1}^l \left[\frac{\partial \varphi(x, y, t)}{\partial x^i} y^i + \frac{\partial \varphi(x, y, t)}{\partial y^i} [-\lambda_i y_i + f_i(x) + b_i u_i] \right] \\ &H(x, y, \nabla_{x,y}\varphi, u, t) \end{aligned}$$

$$\tilde{u}(x, y, \nabla_{x,y}\varphi, t) \in \text{Arg max}_u H(x, y, \nabla_{x,y}\varphi, u, t),$$

$$H(x, y, \nabla_{x,y}(\varphi, t), t) = H(x, y, \nabla_{x,y}\varphi, \tilde{u}(x, y, \nabla_{x,y}\varphi, t), t),$$

$$\psi_s(t) = \nabla_{x,y}\varphi(x, y, t)|_{x=x_s(t), y=y_s(t)}$$

Step 2. We solve Cauchy problem (2) at $u = \tilde{u}(x, y, \nabla_{x,y}\varphi, t)$ and find function of state $\{x_{s+1}(t), y_{s+1}(t)\}$ and control function

$$u_{s+1}(t) = \tilde{u}(x, y, \nabla_{x,y}\varphi_s(x, y, t), t)|_{x=x_s(t), y=y_s(t)}$$

Thus, we find new functional approximation of control state $\{x_{s+1}(t), y_{s+1}(t), u_{s+1}(t)\}$, for which the inequality is true:

$$J(x_{s+1}(t), y_{s+1}(t), u_{s+1}(t)) \leq J(x_s(t), y_s(t), u_s(t)).$$

Application of iterative algorithm to solve the problem of optimal control of steam turbines' capacity.

One of the models describing the transient processes in electrical system is the following system of differential equations [1, 2]:

$$\begin{aligned} \frac{d\delta_i}{dt} &= S_i, \quad H_i \frac{dS_i}{dt} = -D_i S_i - E_i^2 Y_{ii} \sin \alpha_{ii} - P_i \sin(\delta_i - \alpha_i) - \\ &- \sum_{j=1, j \neq i}^l P_{ij} \sin(\delta_{ij} - \alpha_{ij}) + u_i, \quad i \in \overline{1, l}, \quad t \in (0, T), \end{aligned} \quad (10)$$

$$\delta_{ij} = \delta_i - \delta_j, P_i = E_i U Y_{i,n+1}, P_{ij} = E_i E_j Y_{ij},$$

where δ_i is an angle of rotor deflection of i -alternator towards some synchronous roll axis (roll axis of constant voltage bus, which makes rotation at a speed of 50 rpm/sec.; S_i – slip of i -alternator; H_i – an inertia constant of i -alternator; $u_i = P_i$ – mechanical outputs, which fed to alternator; E_i – EMF of i -alternator; Y_{ij} – mutual conductance of system branches i – and j ; $U = const$ is tension in constant voltage bus; $Y_{i,n+1}$ characterizes connection (conductivity) of i – alternator with constant voltage bus; $D_i = const \geq 0$ – mechanical dumping; a_{ij}, a_i – constant values with active resistance influence in armature alternator circuits.

The complexity of the model’s analysis (10) is in taking account $a_{ij}, a_{ij} = a_{ji}, i, j = \overline{1, l}$. Because $\delta_{ij} = -\delta_{ji}$, then the model (10) is not a conservative; you cannot build a Lyapunov function for it in the form of the first integral. The system is called positional model.

Let the state variable and control variable in the established post-emergency mode are equal to:

$$S_i = 0, \delta_i = \delta_i^F, u_i = u_i^F, i = \overline{1, l}. \tag{11}$$

To obtain the system of perturbed motion let us pass on to equations in fluctuations, supposing that:

$$S_i = \Delta S_i, \delta_i = \delta_i^F + \Delta \delta_i, u_i = u_i^F + \Delta u_i, i = \overline{1, l}. \tag{12}$$

Next, for the convenience of the variables $\Delta S_i, \Delta \delta_i, \Delta u_i$, again symbolizing S_i, δ_i, u_i from(11) we get:

$$\frac{d\delta_i}{dt} = S_i, \frac{dS_i}{dt} = \frac{1}{H_i} [-D_i S_i - f_i(\delta_i) - N_i(\delta) + M_i(\delta) + u_i],$$

$$i = \overline{1, l}, t \in (0, T), \tag{13}$$

where

$$f_i(\delta_i) = P_i [\sin(\delta_i + \delta_i^F - \alpha_i) - \sin(\delta_i^F - \alpha_i)],$$

$$N_i(\delta) = \sum_{j=1, j \neq i}^l \overline{N}_{ij}(\delta_1, \dots, \delta_l) = \sum_{j=1, j \neq i}^l \Gamma_{ij}^1 [\sin(\delta_{ij} + \delta_{ij}^F) - \sin \delta_{ij}^F],$$

$$M_i(\delta) = \sum_{j=1, j \neq i}^l \overline{M}_{ij}(\delta_1, \dots, \delta_l) = \Gamma_{ij}^1 [\cos(\delta_{ij} + \delta_{ij}^F) - \cos \delta_{ij}^F],$$

$$\Gamma_{ij}^1 = P_{ij} \cos \alpha_i, \Gamma_{ij}^2 = P_{ij} \sin \alpha_i, P_{ij} = P_{ji}, \Gamma_{ij}^k = \Gamma_{ji}^k, k = 1, 2.$$

The control will be searched in the form of:

$$u_i = v_i - M_i(\delta), i = \overline{1, l}, \tag{14}$$

where v_i to be determined.

It is required to minimize the functional

$$J(v) = J(v_1, \dots, v_l) = 0.5 \sum_{i=1}^l \int_0^T (w_{si} S_i^2 + w_{vi} v_i^2) dt + \Lambda(\delta(T), S(T)), \tag{15}$$

Under the condition (13)-(14), where w_{si}, w_{vi} — positive constants of weight coefficients;

$f_i(\delta_i) - 2\pi$ continuously differentiable periodic function; $N_i(S) \sim 2mm$ - continuously differentiable periodic function towards δ_{ij} ; for $N_i(\delta)$ the condition of the integrability of the type (3) is accomplished; T - the duration of the transition process is considered as given. In addition, the initial conditions have been given:

$$\delta_i(0) = \delta_{i0}, S_i(0) = S_{i0}, i = \overline{1, l}, \tag{16}$$

Final value of the system status $\delta_i(T)$, $S_i(T)$ is not known in advance, they should be determined by solving optimal control problem (13) - (16).

To solve this problem for electric power systems used Krotov theorem on sufficient optimality conditions [1,2]. As a result, we obtain the following theorem.

Lemma 2. In order to manage $v_i^0 = -\frac{S_i}{w_{vi}}$, $i = \overline{1, l}$ and the relevant decision $\{\delta^0, S^0\}$ systems (13) to be optimum, it is also necessary and enough that

$$\Lambda(\delta(T), S(T)) = -\varphi(\delta(T), S(T)), w_{si} = 2D_i + \frac{1}{w_{vi}} > 0, i = \overline{1, l},$$

$$\varphi(\delta, S) = 0.5 \sum_{i=1}^l \left[H_i S_i^2 + \int_0^{\delta_i} f_i(\delta_i) d\delta_i \right] + \sum_{i=1}^l \int_0^{\delta_i} N_i(\delta_1, \dots, \delta_{i-1}, \xi_i, \delta_{i+1}, \dots, \delta_l) d\xi_i$$

where φ – Bellman-Krotov's function, besides,

$$J(v^0) = \min_v J(v) = -\varphi(\delta^0, S^0)$$

In conditions of lemma 2 assumptions (8) from lemma 1 take the form of:

$$\varphi_{\delta_i} S_i = \frac{\varphi S_i}{H_i} [f_i(\delta_i) + N_i(\delta)], \quad \text{т. е. } \varphi_{S_i} = H_i S_i, \varphi_{\delta_i} = f_i(\delta_i) + N_i(\delta), i = \overline{1, l}.$$

3 Numerical example. The optimal motion control of two-unit electric power system.

In the system (10) we take $i = 1, 2$, and assume that the mechanical damping is not available, i.e. the coefficients D_1, D_2 are equal to zero. According to the values (10)-(16), the optimal control problem takes the form of [3]:

$$J(u) = J(u_1, u_2) = 0.5 \sum_{i=1}^2 \int_0^T (10S_i^2 + 0.1v_i^2) dt + 0.5(\delta^2(T), S^2(T)), \quad (17)$$

$$\frac{d\delta_i}{dt} = S_i, \frac{dS_i}{dt} = \frac{1}{H_i} [-f_i(\delta_i) - N_i(\delta) + v_i], i = 1, 2 \quad (18)$$

where $f_i(\delta_i) = P_i [\sin(\delta_i + \delta_i^F - \alpha_i) - \sin(\delta_i^F - \alpha_i)], i = 1, 2,$

$$N_1(\delta) = \Gamma_1 [\sin(\delta_{12} + \delta_{12}^F) - \sin \delta_{12}^F],$$

$$M_1(\delta) = \Gamma_2 [\cos(\delta_{12} + \delta_{12}^F) - \cos \delta_{12}^F],$$

$$\delta_{12}^F = \delta_1^F - \delta_2^F, \quad \Gamma_1 = P_{12} \cos \alpha_{12},$$

$$\Gamma_2 = P_{12} \sin \alpha_{12}, \quad \delta_{12} = \delta_1 - \delta_2, \delta_{21} = -\delta_{12}$$

Numerics of the system (30):

α_1	α_2	H_1	H_2	P_1	P_2	P_{12}	δ_1^F	δ_2^F	α_{12}
-0,052	-0,104	2135	1256	0,85	0,69	0,9	0,827	0,828	-0,078

and initial data:

$$\delta_1(0) = 0.18; \delta_2(0) = 0.1; S_1(0) = 0.001; S_2(0) = 0.002$$

The results are shown in figures 1 and 2. Herewith, the value of a functional (17) has been reduced to the value $\approx 0,006865$.

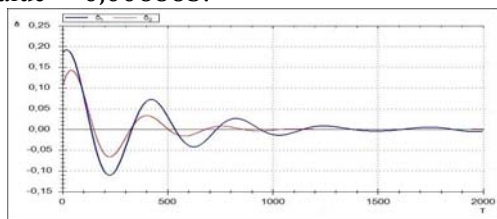


Figure 1 - Functions δ_1, δ_2 with control,

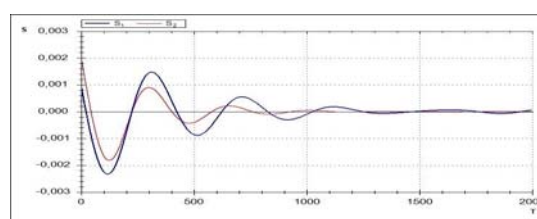


Figure 2 - Functions S_1, S_2 with control

We use the 4-th order Adams-Bashford, Adams-Moulton and Runge-Kutta methods for more accurate results.

Adams-Bashford (A-B) method:

$$y_{n+4} = y_{n+3} + \frac{h}{24}(55 f(t_{n+3}, y_{n+3}) - 59 f(t_{n+2}, y_{n+2}) + 37 f(t_{n+1}, y_{n+1}) - 9 f(t_n, y_n)), \frac{251}{720} h^5(\eta)$$

Adams-Moulton (A-M) method:

$$y_{n+4} = y_{n+3} + \frac{h}{24}(9 f(t_{n+4}, y_{n+4}) + 19 f(t_{n+3}, y_{n+3}) - 5 f(t_{n+2}, y_{n+2}) + f(t_{n+1}, y_{n+1})), - \frac{19}{720} h^5(\eta).$$

Runge-Kutta (R-K) method:

$$y_{n+1} = y_n + \frac{h}{6}(k_1 + 2k_2 + 2k_3 + k_4),$$

Comparison of the used methods is shown below:

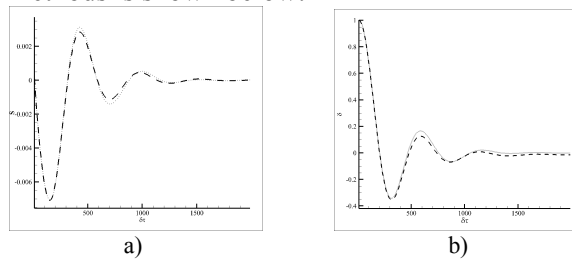


Figure 3 – methods of A-B (line - - -), A-M (line ···) of the 4-th order
a) time change of S ; b) time change of δ

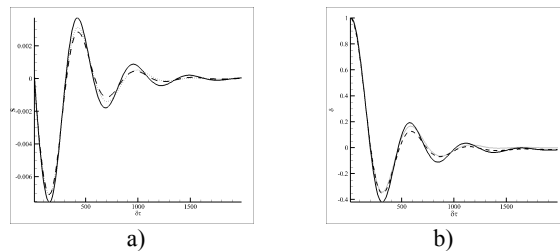


Figure 4 – methods of A-B (line- - -), A-M (line ···) and Euler (line -) of the 4-th order
a) time change of S ; b) time change of δ

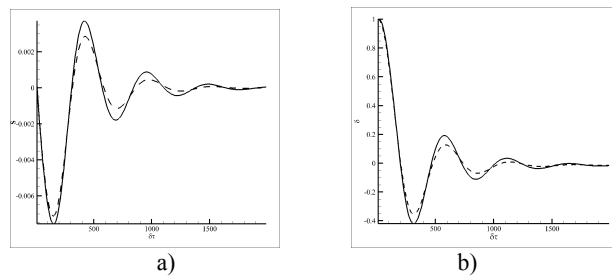


Figure 5 – methods of R-K (line - - -) and Euler (line -) of the 4-th order
a) time change of S ; b) time change of δ

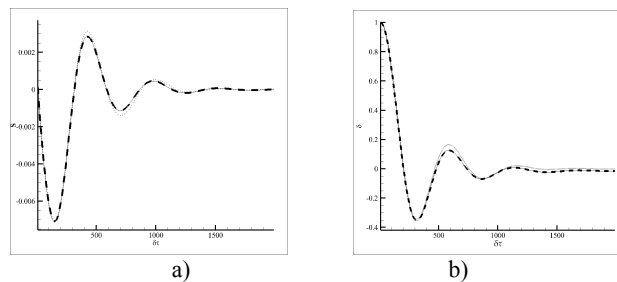


Figure 6 – methods of A-B (line - - -), A-M (line ···) and R-K (line-) of the 4-th order
a) time change of S ; b) time change of δ

According to the obtained results it is clear that the increase of more than 4 is not necessary, as they equally converge to zero.

For this task the Adams-Bashford and Runge-Kutta methods converge to zero faster than when using the method of Adams-Moulton. It allows to reduce time and speed up the process of determining emergency situation. Since the Adams-Moulton method is implicit and requires the solution of the "historical" values, which takes computation time.

Conclusions

The paper deals with solution of optimal control of nonlinear system of ordinary differential equations in two different cases. The studied model, in particular, describes management processes in electric power systems. The proposed methods for solving hold to the extreme tasks expansion principle, based on sufficient optimality conditions of V.F. Krotov. The numerical experiments have shown sufficient efficacy implemented algorithms.

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ЭЛЕКТР ЭНЕРГЕТИКАЛЫҚ ЖҮЙЕЛЕРДІ ОҢТАЙЛЫ БАСҚАРУ

Аннотация. Зерттеудің өзектілігі күрделі электр-энергетикалық кешендердің жаңартылуына және динамикасын зерттеуге негізделген. Индустриалды қоғамның заманауи дамуы қуатты электр-энергетикалық кешендердің құрылуын айқындап, электр-энергиясының тұрақты және үзіліссіз артуын қамтамасыз етеді.

Турбогенераторлардың басым көпшілігі және жиі байланысты энергетикалық объектілерді қамтитын күрделі электр-энергетикалық жүйелердің жұмысының орнықтылығы мен қауіпсіздігін қамтамасыз ету және тиімді басқару мәселелерін зерттеудің өзектілігі мен практикалық құндылығының маңызы зор.

Турбогенераторлар мен күрделі түрде тығыз байланысты энергетикалық блоктардан тұратын заманауи электр-энергетикалық кешеннің математикалық моделі қарапайым дифференциалдық теңдеулердің сызықсыз жүйесін құрайды.

Тиімді басқару теориясы Л.С.Портнягиннің максимум принципі мен Р. Беллманның динамикалық бағдарламалау әдісіне негізделеді. Максимум принципі экстремальды есепті қарапайым дифференциалдық теңдеулердің арнайы жүйесінің шешіміне әкелсе, ал динамикалық бағдарламалау әдісі жеке туындылық есептің шешімін табатыны белгілі. Көптеген жағдайларда бұл есептердің нақты шешімін табу жеткілікті деңгейде қиын.

Бұл мақалада электр энергетикалық жүйелерді оңтайлы басқару мәселелерін зерттеу қарастырылған. Итерациялық алгоритмді қолдана отырып, күрделі электр энергетикалық жүйелерін басқарудың оңтайлы есептерінің сандық шешімі көрсетілген. Екі түрлі жағдайда қарапайым дифференциалдық теңдеулердің сызықты емес жүйесін оңтайлы басқаруды шешу мәселелері қарастырылады. Ұсынылған шешу әдістері В.Ф. Кротовтың оңтайлы болуы үшін жеткілікті жағдайларға негізделген экстремалды мәселелерді жалғастыру қағидасын ұстанады. Оңтайлы басқару мәселелерінің ерекше жағдайы қарастырылады. Сандық тәжірибелер орындалған алгоритмдердің тиімділігін көрсетті. Ұсынылған сандық мысалда екі жүйелі электр энергиясының оңтайлы қозғалысын басқару мәселесі графикалық түрде көрсетілген.

Түйін сөздер: оңтайлы басқару, электр энергетикалық жүйелер, итерациялық алгоритм.

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ОПТИМАЛЬНОЕ УПРАВЛЕНИЕ ЭЛЕКТРОЭНЕРГЕТИЧЕСКИМИ СИСТЕМАМИ

Аннотация. Актуальность исследования обусловлена необходимостью модернизации и исследования динамики сложных электроэнергетических комплексов. Современное развитие индустриального общества требует постоянного и непрерывного роста производства электроэнергии, для обеспечения которой создаются мощные электроэнергетические комплексы.

Особую актуальность и практическое значение представляют исследования оптимального управления, обеспечения безопасности и устойчивости работы сложных электроэнергетических систем, состоящих из большого числа турбогенераторов и многосвязных энергетических объектов.

Математическая модель современной энергетической установки, состоящей из турбогенераторов и сложно связанных энергоблоков, образует нелинейную систему простых дифференциальных уравнений.

Теория эффективного управления основана на принципе максимума Л. С. Понтрягина и метода динамического программирования Беллмана. Известно, что принцип максимума приводит к решению экстремальной задачи к специальной системе простых дифференциальных уравнений, а метод динамического программирования приводит к решению индивидуальной задачи о продукте. Во многих случаях довольно сложно найти точное решение этих проблем.

В данной статье рассматриваются вопросы исследования оптимального управления электроэнергетическими системами. Показано численное решение задач оптимального управления сложными электроэнергетическими системами с использованием итерационного алгоритма. Также рассматриваются вопросы решения оптимального управления нелинейной системой обыкновенных дифференциальных уравнений в двух разных случаях. Предложенные методы решения следуют принципу продолжения экстремальных задач на основе достаточных условий оптимальности В. Ф. Кротова. Рассмотрен частный случай задач оптимального управления. Численные эксперименты показали достаточную эффективность реализованных алгоритмов. Задача оптимального управления движением двухсистемной электроэнергетической системы графически проиллюстрирована на предлагаемом численном примере.

Ключевые слова: оптимальное управление, электроэнергетические системы, итерационный алгоритм.

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