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EQUATIONS OF PLANETARY SYSTEMS MOTION

Abstract. The study of the dynamically evolution of planetary systems is very actually in relation with findings of exoplanet systems. N free spherical bodies problem is considered in this paper, mutually gravitating according to Newton's law, with isotropically variable masses as a celestial-mechanical model of non-stationary exoplanetary systems. The dynamic evolution of planetary systems is learned, when evolution's leading factor is the masses' variability of gravitating bodies themselves. The laws of the bodies' masses varying are assumed to be known arbitrary functions of time. When doing so the rate of varying of bodies' masses is different. The planets' location is such that the orbits of planets don't intersect. Let us treat this position of planets is preserve in the evolution course. The motions are researched in the relative coordinates system with beginning in the center of the parent star, axes that are parallel to corresponding axes of the absolute coordinates system. The canonical perturbation theory is used on the base aperiodic motion over the quasi-canonical cross-section. The bodies evolution is studied in the osculating analogues of the second system of canonical Poincare elements. The canonical equations of perturbed motion in analogues of the second system of canonical Poincare elements are convenient for describing the planetary systems dynamic evolution, when analogues of eccentricities and analogues of inclinations of orbital plane are sufficiently small. It is noted that to obtain an analytical expression of the perturbing function main part through canonical osculating Poincare elements using computer algebra is preferably. If in expansions of the main part of perturbing function is constrained with precision to second orders including relatively small quantities, then the equations of secular perturbations will obtained as linear non-autonomous system. This circumstance meaningful makes much easier to study the non-autonomous canonical system of differential equations of secular perturbations of considering problem.

Keywords: planetary systems, variable mass, Poincare elements, theory of perturbations, evolution equations.

1. Introduction. In relation with findings of exoplanet systems, the study of the dynamically evolution of planetary systems is very actually. Observations materials are wealthy [1-3], especially the study of planetary systems in the stage of non-stationary is represents of interesting, when leading factor of evolution is variability of masses of graviting bodies [4-7].

In this paper, n free spherical bodies problem is considered, mutually gravitating according to Newton's law, with isotropically variable masses. The laws of the masses varying will be considered to be known. On the base aperiodic motion over the quasi-canonical cross-section the canonical perturbation theory is used [7-8]. The motions are researched in the relative coordinates system, in the analogues of the second system of canonical Poincare elements.

2. Problem statement and differential equations of motion in the relative coordinates system. The motion of planetary systems is considered, consisting of $n+1$ spherical bodies with variable masses mutually gravitating according to Newton's law. The following notations are entered: T_0 – the parent star of planetary system, T_i, $(i=1,2,...,n)$ – planets. The motions are studied in the relative coordinates system with the beginning in the center of the parent star T_0, axes that are parallel to corresponding axes of the absolute coordinates system.

The planets' location is such that T_i is inner planet relative to planets T_{i+1}, but outer one relative to T_{i-1}. Let us treat that this position of planets in the evolution course is preserve.

The masses of bodies are changed isotropically over time and laws of variable of masses are assumed to be known

$$m_0 = m_0(t), \quad m_1 = m_1(t), \quad ..., \quad m_n = m_n(t) \tag{2.1}$$

Let the rate of masses varying is different

$$\frac{\dot{m}_0}{m_0} \neq \frac{\dot{m}_i}{m_i}, \qquad \frac{\dot{m}_i}{m_i} \neq \frac{\dot{m}_k}{m_k}, \qquad i,k = 1,2,..,n, \qquad i \neq k, \tag{2.2}$$

The motion equations of n planets in the relative coordinates system with isotropically variable masses can be written as [7,9]

$$\ddot{\vec{r}}_i = -f\frac{(m_0 + m_i)}{r_i^3}\vec{r}_i + f\sum_{k=1}^{n}{}' m_k\left(\frac{\vec{r}_k - \vec{r}_i}{\Delta_{ik}^3} - \frac{\vec{r}_k}{r_k^3}\right), \ (i = 1,2,...,n), \quad (\mathrm{k} = 1,2,...,n) \tag{2.3}$$

where Δ_{ik} mutual distances of the center of spherical bodies

$$\Delta_{ik} = \sqrt{(x_k - x_i)^2 + (y_k - y_i)^2 + (z_k - z_i)^2} = \Delta_{ki}\ , \tag{2.4}$$

f – the gravitational constant, $m_0 = m_0(t)$ – mass of the parent star, $m_i = m_i(t)$ – mass of planet T_i, $\vec{r}_i(x_i, \mathrm{y}_i, \mathrm{z}_i)$ – radius-vector of the center of spherical bodies, sign "stroke" in the summation denotes that the $i \neq k$.

3. The motion equations in osculating elements. Equations of the motion (2.3) are rewritten in the form

$$\ddot{\vec{r}}_i + f\frac{(m_0 + m_i)}{r_i^3}\vec{r}_i - \frac{\ddot{\gamma}_i}{\gamma_i}\vec{r}_i = \vec{F}_i\ ,\ \gamma_i = \frac{m_0(t_0) + m_i(t_0)}{m_0(t) + m_i(t)} = \gamma_i(t), \tag{3.1}$$

where t_0 – is initial moment of time,

$$\vec{F}_i = grad_{\vec{r}_i} W_i\,,$$
$$W_i = W_{ci} + W_{gi}\,, \tag{3.2}$$

$$W_{gi} = f\sum_{k=1}^{n}{}' m_k\left(\frac{1}{\Delta_{ik}} - \frac{\vec{r}_i \cdot \vec{r}_k}{r_k^3}\right), \ W_{ci} = -\frac{\ddot{\gamma}_i}{2\gamma_i}r_i^2\,. \tag{3.3}$$

Obtained form of relative motion equations (3.1) is convenient to use perturbations theory formulated to such non-stationary systems [7].

When doing so expressions (3.2) are the perturbing forces. If the perturbing forces are equal to zero, then obtain integrable unperturbed motions.

At $W_i = 0$ the equations (3.1) describe aperiodic motion over the quasi-canonical cross-section [7,8]

$$\ddot{\vec{r}}_i + f\frac{(m_0 + m_i)}{r_i^3}\vec{r}_i - \frac{\ddot{\gamma}_i}{\gamma_i}\vec{r}_i = 0 \tag{3.4}$$

The solution of differential equations (3.4) is similar to the solution of the classical two body problem with constant masses

$$\begin{aligned} x_i &= \gamma_i\rho_i\left[\cos u_i \cdot \cos\Omega_i - \sin u_i \cdot \sin\Omega_i \cdot \cos i_i\right], \\ y_i &= \gamma_i\rho_i\left[\cos u_i \cdot \sin\Omega_i + \sin u_i \cdot \cos\Omega_i \cdot \cos i_i\right], \\ z_i &= \gamma_i\rho_i \sin u_i \cdot \sin i_i, \quad r_i = \gamma_i\rho_i, \quad u_i = \theta_i + \omega_i, \end{aligned} \tag{3.5}$$

$$\rho_i = \frac{a_i\left(1-e_i^2\right)}{1+e_i\cos\theta_i}, \tag{3.6}$$

where u_i – analogue of the latitude argument, θ_i – analogue of the true anomaly. The solutions (3.5)-(3.6) will be used as initial unperturbed motion. The values

$$a_i,\ e_i,\ i_i,\ \omega_i,\ \Omega_i \tag{3.7}$$

are analogues of known Kepler elements. When doing so, a_i – analogue of a semimajor axis, e_i – analogue of eccentricity, ω_i – analogue of the pericenter argument, i_i – analogue of inclinations of the orbit, Ω_i – analogue of the longitude of an ascending node.

In the case of quasi-elliptic motion $e_i<1$ the dependence of analogues of the mean anomaly M_i on time

$$M_i = n_i\left[\phi_i(t)-\phi_i(\tau_i)\right], \tag{3.8}$$

are determined taking into account the laws of masses variation. Here n_i – the analogue of mean motion, μ_{i0} – gravitational parameter

$$n_i = \frac{\sqrt{\mu_{i0}}}{a_i^{3/2}} = const,\quad \mu_{i0} = f\left[m_0\left(t_0\right)+m_i\left(t_0\right)\right]. \tag{3.9}$$

When doing so, $\phi_i(t)$ – the primary function of the values

$$\frac{1}{\gamma_i^2\left(t\right)} = \left(\frac{m_0(t)+m_i(t)}{m_0(t_0)+m_i(t_0)}\right)^2. \tag{3.10}$$

Correspondingly $\phi_i(\tau_i)$ is a dynamically element, analogue of the pericenter passage time. By τ_i – is denoted passage time through the pericenter. We emphasize that in unperturbed motion mean angular velocity is variable and depends on laws of masses variability of corresponding bodies.

$$\dot{M}_i = n_i\left(\frac{1}{\gamma_i^2(t)}\right) = n_i\left(\frac{m_0(t)+m_i(t)}{m_0(t_0)+m_i(t_0)}\right)^2. \tag{3.11}$$

In unperturbed motion, formally mathematically, the Kepler equations are occurs, which allows to find coordinates and velocities as functions of time.

In the case of quasi-elliptic motion $(e_i<1)$, regular integration of the differential equation of unperturbed motion (3.7) can be defined by the following six elements of quasi-elliptic motion

$$a_i,\ e_i,\ i_i,\ \omega_i,\ \Omega_i,\ \phi_i(\tau_i). \tag{3.12}$$

In work [7] a corresponding perturbation theory is constructed, which will be widely used in this paper, by using elements (3.12) as unperturbed.

For our purposes analogues of the second system of canonical Poincare elements are prefered

$$\Lambda_i,\ \lambda_i,\ \xi_i,\ \eta_i,\ p_i,\ q_i, \tag{3.13}$$

which are entered according to the formulas

$$\Lambda_i = \sqrt{\mu_{i0}}\sqrt{a_i},\quad \lambda_i = l_i + \pi_i, \tag{3.14}$$

$$\xi_i = \sqrt{2\sqrt{\mu_{i0}}\sqrt{a_i}\left(1-\sqrt{1-e_i^2}\right)}Cos\pi_i,\quad \eta_i = -\sqrt{2\sqrt{\mu_{i0}}\sqrt{a_i}\left(1-\sqrt{1-e_i^2}\right)}Sin\pi_i, \tag{3.15}$$

$$p_i = \sqrt{2\sqrt{\mu_{i0}}\sqrt{a_i}\sqrt{1-e_i^2}\left(1-Cosi_i\right)}Cos\Omega_i,\quad q_i = -\sqrt{2\sqrt{\mu_{i0}}\sqrt{a_i}\sqrt{1-e_i^2}\left(1-Cosi_i\right)}Sin\Omega_i. \tag{3.16}$$

where

$$l_i = M_i = n_i\left[\phi_i(t)-\phi_i(\tau_i)\right],\quad \pi_i = \Omega_i + \omega_i, \tag{3.17}$$

The differential equations of n spherical bodies motion in the osculating analogues of the second system of Poincare variable (3.14)-(3.16) have canonical form

$$\dot{\Lambda}_i = \frac{\partial R_i^*}{\partial \lambda_i}, \quad \dot{\xi}_i = \frac{\partial R_i^*}{\partial \eta_i}, \quad \dot{p}_i = \frac{\partial R_i^*}{\partial q_i},$$
$$\dot{\lambda}_i = -\frac{\partial R_i^*}{\partial \Lambda_i}, \quad \dot{\eta}_i = -\frac{\partial R_i^*}{\partial \xi_i}, \quad \dot{q}_i = -\frac{\partial R_i^*}{\partial p_i}, \tag{3.18}$$

where the Hamilton functions

$$R_i^* = \frac{\mu_{i0}^2}{2\Lambda_i^2} \cdot \frac{1}{\gamma_i^2(t)} + W_i(t, \Lambda_i, \xi_i, p_i, \lambda_i, \eta_i, q_i) \tag{3.19}$$

The canonical equations of perturbed motion (3.18) are convenient for describing the dynamic evolution of planetary systems, when analogues of eccentricities and analogues of inclinations of orbital plane are sufficiently small.

$$e_i \ll 1, \quad i_i \ll 1. \tag{3.20}$$

The canonical equations of motion (3.18) rewrite in the form

$$\dot{\lambda}_i = -\frac{\partial R_i^*}{\partial \Lambda_i} = \frac{\mu_{i0}^2}{\gamma_i^2 \Lambda_i^3} - \frac{\partial W_i}{\partial \Lambda_i}, \qquad \dot{\Lambda}_i = \frac{\partial R_i^*}{\partial \lambda_i} = \frac{\partial W_i}{\partial \lambda_i},$$
$$\dot{\eta}_i = -\frac{\partial R_i^*}{\partial \xi_i} = -\frac{\partial W_i}{\partial \xi_i}, \qquad \dot{\xi}_i = \frac{\partial R_i^*}{\partial \eta_i} = \frac{\partial W_i}{\partial \eta_i}, \tag{3.21}$$
$$\dot{q}_i = -\frac{\partial R_i^*}{\partial p_i} = -\frac{\partial W_i}{\partial p_i}, \qquad \dot{p}_i = \frac{\partial R_i^*}{\partial q_i} = \frac{\partial W_i}{\partial q_i}.$$

At $W_i = 0$ it can be seen from equations (3.21) that the canonical variables $\Lambda_i, \eta_i, \xi_i, q_i, p_i$ remain constant, and the element λ_i – mean longitude is an increasing function of time.

4. The expansion of the perturbing functions. To write explicitly the right-hand sides of the perturbed motion equations (3.21), it is necessary to Express the perturbing function (3.2)-(3.3) in terms of the osculating elements (3.14) - (3.16). The expression of the value W_{ci} through osculating elements is simple, and its explicit analytical form is known [7]. The main difficulties represent the expansion into a series of the force function of the Newtonian interaction of bodies

$$W_{gi} = f\sum_{k=1}^{n}{}' m_k \left(\frac{1}{\Delta_{ik}} - \frac{\vec{r}_i \cdot \vec{r}_k}{r_k^3} \right). \tag{4.1}$$

through osculating elements (3.14)-(3.16).

It is advisable to emphasize the main and indirect parts in the expression of the perturbing function (4.1)

$$W_{gi} = f\sum_{k=1}^{n}{}' m_k \left(\frac{1}{\Delta_{ik}} - \frac{\vec{r}_i \cdot \vec{r}_k}{r_k^3} \right) = f\sum_{k=1}^{n}{}' m_k \left(\frac{1}{\Delta_{ik}} \right) - f\sum_{k=1}^{n}{}' m_k \left(\frac{x_i x_k + y_i y_k + z_i z_k}{r_k^3} \right) =$$
$$= f\sum_{k=1}^{n}{}' m_k \left(\frac{1}{\Delta_{ik}} \right) - f\sum_{k=1}^{n}{}' m_k \left(\frac{r_i \cdot r_k \cdot Cos\psi_{ik}}{r_k^3} \right) = f\sum_{k=1}^{n}{}' m_k \left(\frac{1}{\Delta_{ik}} \right) - f\sum_{k=1}^{n}{}' m_k \left(r_i \cdot \left(\frac{1}{r_k^2} \right) \cdot Cos\psi_{ik} \right). \tag{4.2}$$

We denote the main and indirect parts of the perturbing function (4.1)

$$W_{gi,main} = f\sum_{k=1}^{n}{}' m_k \left(\frac{1}{\Delta_{ik}} \right), \; W_{gi,ind} = -f\sum_{k=1}^{n}{}' m_k \left(r_i \cdot \left(\frac{1}{r_k^2} \right) Cos(\psi_{ik}) \right). \tag{4.3}$$

The indirect part of the perturbing function (4.3) does not contribute to the expressions of the secular perturbing function as in the classical many body problem [10]. Therefore, it is sufficient to have an analytical expression of the main part of the perturbing function (4.3) in terms of the canonical elements (3.14)-(3.16) to obtain differential equations of secular perturbations in the canonical osculating elements (3.14)-(3.16).

To obtain an analytical expression of the main part of the perturbing function (4.3), through the canonical osculating elements (3.14)-(3.16), it is necessary to have a decomposition in a series of quantities

$$\left(\frac{1}{\Delta_{ik}}\right), \; i,k=1,2,..,n, \quad i\neq k . \tag{4.4}$$

These are very cumbersome and time-consuming algebraic calculations that are performed using computer algebra. In work [8,11,12], such calculations were performed for the two-planet problem of three bodies with variable masses, with precision to second orders including relatively small quantities (3.20).

For the n planetary problem of many body with isotropically varying masses, considered in this paper, the expansion into a series of quantities (4.4) is performed in exactly the same way. However, the calculation volume for a many planet problem is growing, which is natural.

5. The equations of secular perturbations. The equations of secular perturbations that determine the behavior of orbital parameters over long time intervals are obtained from the equations of motion (3.21) if instead the perturbating functions W_i their secular part is substituted

$$\begin{aligned}
&\dot{\lambda}_i=-\frac{\partial R_i^*}{\partial\Lambda_i}=\frac{\mu_{i0}^2}{\gamma_i^2\Lambda_i^3}-\frac{\partial W_i^{(\sec)}}{\partial\Lambda_i}, && \dot{\Lambda}_i=0,\\
&\dot{\eta}_i=-\frac{\partial R_i^*}{\partial\xi_i}=-\frac{\partial W_i^{(\sec)}}{\partial\xi_i}, && \dot{\xi}_i=\frac{\partial R_i^*}{\partial\eta_i}=\frac{\partial W_i^{(\sec)}}{\partial\eta_i},\\
&\dot{q}_i=-\frac{\partial R_i^*}{\partial p_i}=-\frac{\partial W_i^{(\sec)}}{\partial p_i}, && \dot{p}_i=\frac{\partial R_i^*}{\partial q_i}=\frac{\partial W_i^{(\sec)}}{\partial q_i}.
\end{aligned} \tag{5.1}$$

$$W_i^{(\sec)}=W_{gi}^{(\sec)}+W_{ci}^{(\sec)}, \; W_{gi}^{(\sec)}=W_{gi,\text{гл}}^{(\sec)}. \tag{5.2}$$

Naturally, the following system of canonical equations is split off from the system of differential equations (5.1)

$$\begin{aligned}
&\dot{\eta}_i=-\frac{\partial W_i^{(\sec)}}{\partial\xi_i}, && \dot{\xi}_i=\frac{\partial W_i^{(\sec)}}{\partial\eta_i}, &\\
&&&& i=1,2,..,n,\\
&\dot{q}_i=-\frac{\partial W_i^{(\sec)}}{\partial p_i}, && \dot{p}_i=\frac{\partial W_i^{(\sec)}}{\partial q_i}. &
\end{aligned} \tag{5.3}$$

If, in the expansion into a series of quantities (4.4), we restrict ourselves to second-order accuracy, including relatively small quantities (3.20), then the system of equations (5.3) will turn out to be a linear non-Autonomous system. When doing so approximate formulas for the relationship of various systems of osculating elements as initial assumptions have the form

$$\begin{aligned}
\xi_i=\sqrt{\Lambda_i}e_i\cos\pi_i, \quad \eta_i=-\sqrt{\Lambda_i}e_i\sin\pi_i, \quad \Lambda_i e_i^2=\xi_i^2+\eta_i^2, \quad tg\pi_i=-\eta_i/\xi_i,\\
p_i=\sqrt{\Lambda_i}\sin i_i\cos\Omega_i, \quad q_i=-\sqrt{\Lambda_i}\sin i_i\sin\Omega_i, \quad \Lambda_i\sin^2 i_i=p_i^2+q_i^2, \quad tg\Omega_i=-q_i/p_i.
\end{aligned} \tag{5.4}$$

Then, in turn, the resulting system of canonical equations (5.3) is divided into two separate subsystems (see details in [8]). The first subsystem defines the equations of secular perturbations for eccentric elements. The second subsystem defines the equations of secular perturbations for the oblique elements. The linearity of the system of differential equations (5.3) in the approximation (5.4) significantly facilitates the study of the non-Autonomous canonical system of differential equations of secular perturbations (5.3) of the problem in this formulation.

6. Conclusion. In this paper, the differential equations of secular perturbations for non-stationary n planetary systems with isotropically varying masses in analogs of the second system of canonical Poincare elements are obtained in analytical form.

To obtain the actual expansion of the perturbing function through osculating elements, it is planned to use the analytical computing system "Wolfram Mathematica" [13,14].

The resulting equations will be used to study the effects of mass variability during the evolution of exoplanetary systems. This will take into account the effects of the decrease in the mass of the parent star and the increase in the mass of planets due to the accretion of matter from the remnants of the protoplanetary disk.

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ПЛАНЕТА ЖҮЙЕСІНІҢ ҚОЗҒАЛЫС ТЕҢДЕУЛЕРІ

Аннотация. Экзопланета жүйесінің ашылуына байланысты планета жүйелерінің динамикалық эволюциясын зерттеу өзекті мәселеге айналды. Жұмыста ньютон заңы бойынша өзара гравитацияланатын, нестационарлы экзопланеталық жүйелердің аспан-механикалық моделі ретінде массалары изотропты өзгеретін, еркін сфералық n дене мәселесі қарастырылған. Масса изотропты өзгерген кезде әр денеге әсер ететін реактивті күштердің қосындысы нөлге тең, сол себепті де қозғалыс теңдеуі ықшамдалады. Алайда дене массаларының айнымалылығы әсерінен дифференциалды теңдеулер жүйесі автономды емес түрге келеді әрі бұл мәселені айтарлықтай күрделендіреді. Сондықтан мәселе ұйытқу теориясы әдістерімен зерттеледі. Гравитацияланатын дене массасының өзгерісі эволюцияның жетекші факторы ретінде қарастырылған жағдайда планета жүйелерінің динамикалық эволюциясы зерттеледі. Дене массаларының өзгеру заңдылығы кез келген және белгілі уақыт функциясы ретінде есепке алынады. Сонымен қатар, дене массаларының өзгеру қарқындылығы әртүрлі. Планеталардың орналасуы олардың орбиталары бір-бірімен кездеспейтіндей жағдайда қарастырылған. Эволюция кезінде планеталардың осылай орналасу жағдайы сақталады деп есептейміз. Осындай жолмен ұйытқу теориясының әдістерін қолданудың математикалық дұрыстығы негізге алынады. Планета массалары орталық жұлдыз массасынан әлдеқайда кіші деп қарастырамыз. Планета жүйесінің орталық жұлдызы центрлік дене болып саналады. Жұмыста квазиэллиптикалық қозғалыс зерттеледі. Квазиэллиптикалық қозғалыс жағдайында орташа аномалия аналогының уақытқа тәуелділігі ұйытқымаған қозғалыс кезінде массалардың өзгеру заңдылығы есебінде анықталады. Ұйытқымаған қозғалыс кезінде координаталар мен жылдамдықты уақыт функциясы ретінде анықтауға мүмкіндік беретін Кеплер теңдеуі математикалық тұрғыдан орынды деп саналады. Квазиэллиптикалық жағдайда ұйытқымаған қозғалыстың дифференциалдық теңдеуін тұрақты интегралдау квазиэллиптикалық қозғалыстың алты элементтің сәйкес аналогтарымен анықталады. Жұмыста Пуанкаре канондық элементтердің екінші жүйе аналогы негізінде канондық ұйытқу теориясы кең қолданылады. Дене эволюциясы Пуанкаре канондық элементтерінің екінші жүйесінің лездік аналогтарында зерттеледі. Эксцентриситет аналогтары мен орбита жазықтығының көлбеулік аналогтары жеткілікті түрде кіші шама болған жағдайда Пуанкаре канондық элементтерінің екінші жүйе аналогтарындағы ұйытқыған қозғалыстың канондық теңдеуі планета жүйелерінің динамикалық эволюциясын сипаттауда ыңғайлы болып саналады. Ұйытқытушы функцияның негізгі бөлігінің аналитикалық өрнегін шексіз қатар ретінде Пуанкаре канондық лездік элементтерінде алу үшін компьютерлік алгебраны қолдану қажеттігі көрсетіледі. Негізінде қатарға жіктеу әдісі ұйытқытушы функцияның негізгі бөлігінің аналитикалық өрнегін кез келген дәлдікте алуға мүмкіндік береді. Егер ұйытқытушы функцияның негізгі бөлігін жіктеуде кіші шамаға қатысты екінші реттік дәлдікті қоса есептеумен шектелсек, онда ғасырлық ұйытқу теңдеулері сызықты автономды емес жүйе болып есептеледі. Осындай жағдайда алынған ғасырлық ұйытқудың канондық теңдеуінің жүйесі екі ішкі жүйеге жіктеледі. Бірінші жүйе ғасырлық ұйытқу теңдеулерін эксцентрицитеттік элементтер үшін анықтайды. Екінші жүйе ғасырлық ұйытқу теңдеулерін обликалық элементтер үшін анықтайды. Осы жағдай қарастырылып отырған мәселенің ғасырлық ұйытқуының дифференциалды теңдеуінің автономды емес канондық жүйесін зерттеуді айтарлықтай жеңілдетеді.

Түйін сөздер: планеталық жүйелер, өзгермелі масса, Пуанкаре элементтері, ұйытқу теориясы, эволюциялық теңдеулер.

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УРАВНЕНИЯ ДВИЖЕНИЯ ПЛАНЕТНЫХ СИСТЕМ

Аннотация. В связи с открытиями экзопланетных систем изучение динамической эволюции планетных систем является весьма актуальной. В настоящей работе рассмотрена задача n свободных сферических тел, взаимогравитирующих по закону Ньютона, с изотропно изменяющимися массами как небесно-механическая модель нестационарных экзопланетных систем. При изотропном изменении масс суммарные реактивные силы, действующие на каждое тело, равны нулю, поэтому уравнение движения упрощается. Однако из-за переменности масс тел система дифференциальных уравнений становится неавтономными, что существенно усложняет задачу. Поэтому проблема исследуется методами теории возмущения. Изучается динамическая эволюция планетных систем, когда ведущим фактором эволюции является переменность масс самих гравитирующих тел. Законы изменении масс тел считаются известными произвольными функциями времени. При этом темп изменения масс тел различный. Расположение планет таково, что орбиты планет не пересекаются. Будем считать, что это положение планет в ходе эволюции сохраняется. Этим обеспечивается математическая корректность применяемых методов теории возмущений. Считается, что массы планет намного меньше, чем масса центрального тела. Центральным телом является родительская звезда рассматриваемой планетной системы. Движения исследованы в относительной системе координат с началом в центре родительской звезды, оси которой параллельны соответствующим осям абсолютной системы координат. Используется каноническая теория возмущений на базе апериодического движения по квазиконическому сечению. В работе рассматривается квазиэллиптическое движение. В случае квазиэлиптического движения зависимость аналогов средней аномалии от времени в невозмущенном движении определяются с учетом законов изменения масс. В невозмущенном движении формально математически имеет место уравнение Кеплера, которое позволяет найти координат и скорости как функции времени. Постоянные интегрирования дифференциального уравнения невозмущенного движения, в случае квазиэллиптического движения, определены шестью элементами квазиэллиптического движения, соответствующими аналогами кеплеровской орбиты. В настоящей работе широко использованы каноническая теория возмущения, на базе аналогов второй системы канонических элементов Пуанкаре. Динамическая эволюция тел также изучается в оскулирующих аналогах второй системы канонических элементов Пуанкаре. Канонические уравнения возмущенного движения в аналогах второй системы канонических элементов Пуанкаре, удобные для описания динамической эволюции планетных систем, когда аналоги эксцентриситетов и аналоги наклонности орбитальной плоскости – достаточно малые величины. Отмечается, что для получения аналитического выражения главной части возмущающей функции в виде бесконечных рядов, выраженные через канонические оскулирующие элементы Пуанкаре, предпочтительно использование компьютерной алгебры. Методика разложения в ряды, в принципе, дает возможность получения аналитического выражения главной части возмущающей функции, с любой заданной точностью. Если в разложение главной части возмущающей функции ограничится с точностью до вторых порядков включительно относительно малых величин, то уравнение вековых возмущений получится линейной неавтономной системой. Тогда полученная система канонических уравнений вековых возмущений разделяется на две отдельные подсистемы. Первая подсистема определяет уравнений вековых возмущений для эксцентрических элементов. Вторая подсистема определяет уравнений вековых возмущений для облических элементов. Это обстоятельство существенно облегчает исследования неавтономной канонической системы дифференциальных уравнений вековых возмущений рассматриваемой проблемы.

Ключевые слова: планетные системы, переменная масса, элементы Пуанкаре, теория возмущений, эволюционные уравнения.

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